

# DECISION PROCEDURE FOR TRACE EQUIVALENCE

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# CONTEXT

## ■ Cryptographic protocols

Most communications take place over a **public** network



### **Cryptographic protocols**

- small programs designed to secure communication (e.g. secrecy)
- use cryptographic primitives (e.g. encryption, signature)

It important to verify their security

# CONTEXT

- Reliable cryptography
- **Correct specification**
- Implementation satisfying the specification

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### **Reachability properties**

- Secrecy, Authentication, ...

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- Reliable cryptography
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## ■ Some security properties

### **Reachability properties**

- Secrecy, Authentication, ...

### **Equivalence properties**

- Anonymity, Privacy, Receipt-Freeness, ...

# CONTEXT

- Formalism



Alice



Bob

# CONTEXT

- Formalism



Alice



Bob



# CONTEXT

## ■ Formalism



Alice



Intruder



Bob

The intruder can

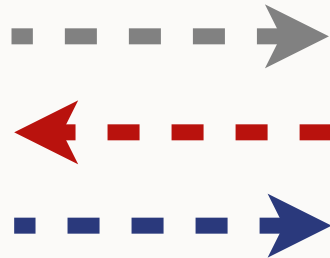
- intercept all messages
- transmit or modify messages
- test equality between messages
- initiate several sessions

# CONTEXT

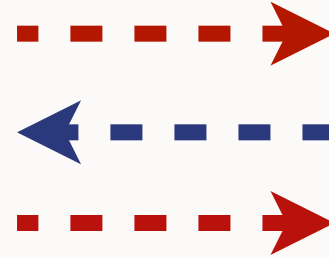
## ■ Formalism



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The intruder can

- intercept all messages
- transmit or modify messages
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# CONTEXT

- Reachability properties : secrecy, authentication,...



Alice



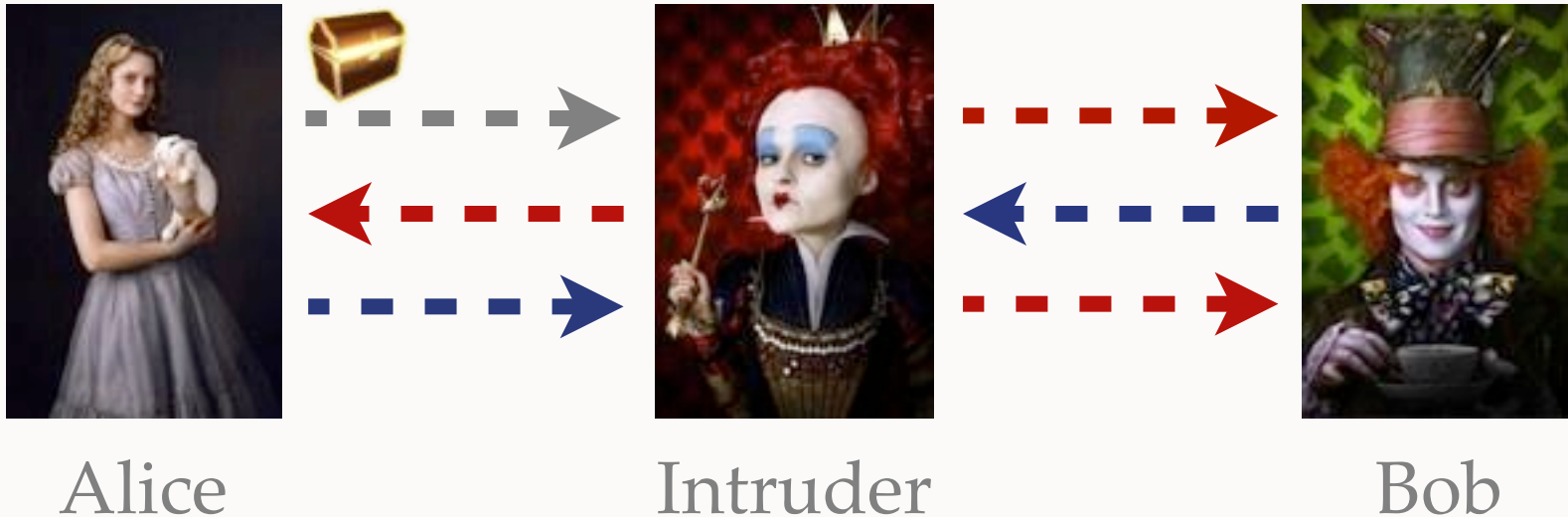
Intruder



Bob

# CONTEXT

- Reachability properties : secrecy, authentication,...

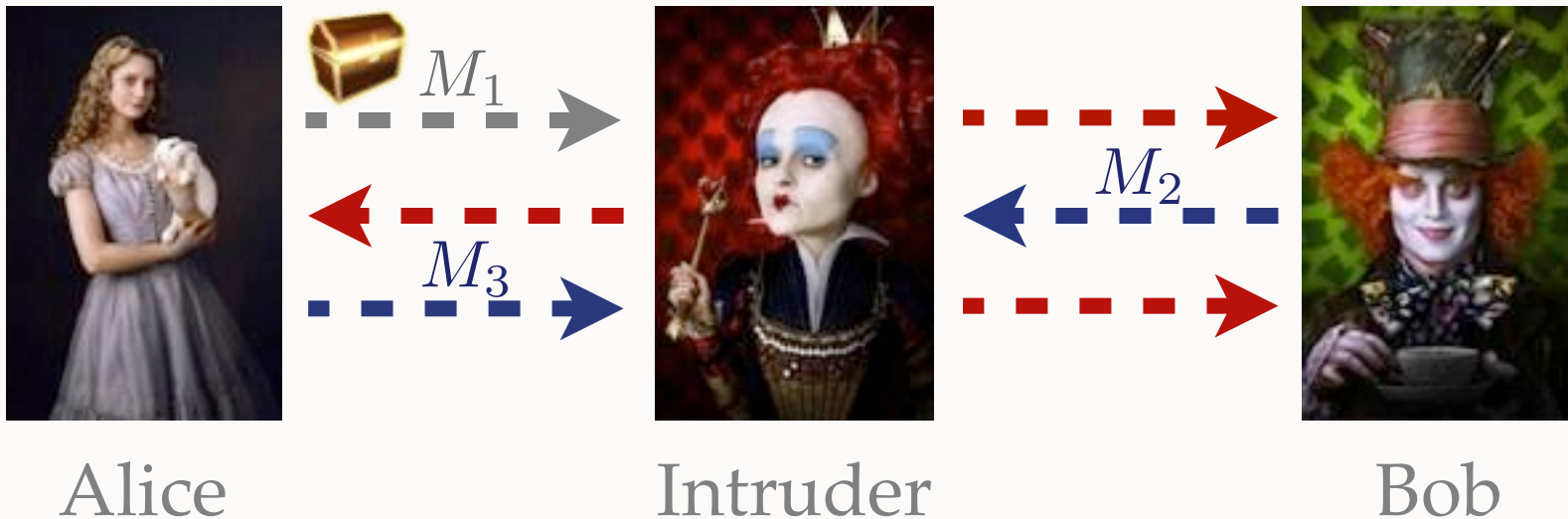


Can the intruder deduce Alice's secret ?

# CONTEXT

- Reachability properties : secrecy, authentication,...

intruder's knowledge :  $M_1 M_2 M_3$  + basic knowledge



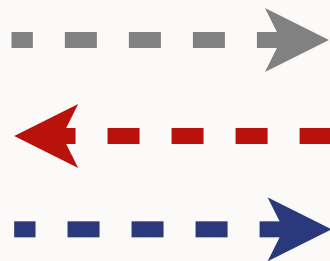
Can the intruder deduce Alice's secret ?

# CONTEXT

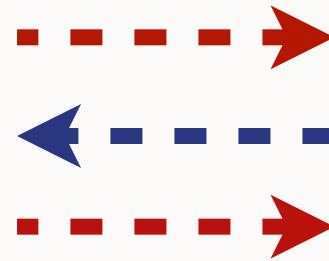
- Equivalence properties : strong secret, anonymity,...



Alice



Intruder



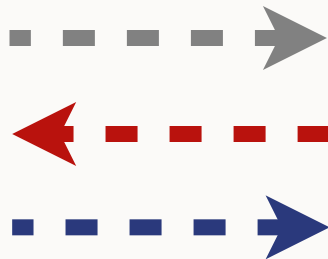
Unknown

# CONTEXT

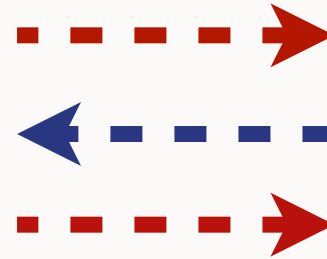
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Alice



Intruder



Unknown

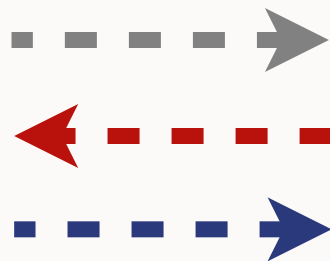
Can the intruder deduce the unknown's identity ?

# CONTEXT

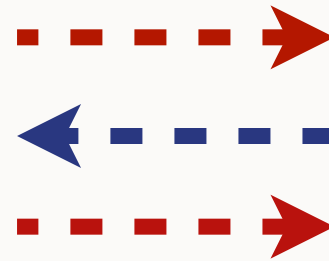
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Alice



Intruder



Unknown

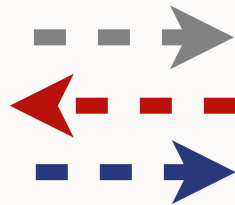


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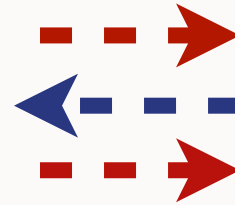
- Equivalence properties : strong secret, anonymity,...



Alice



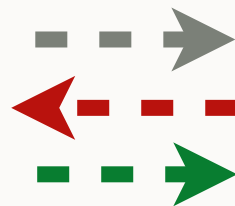
Intruder



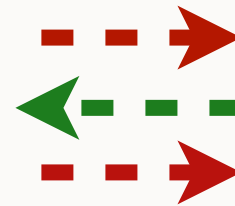
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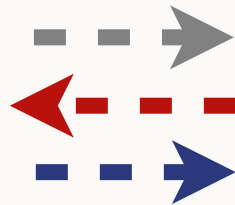
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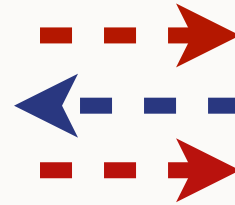
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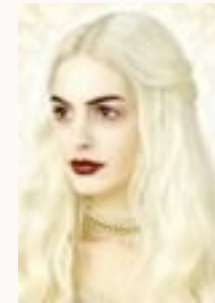
Alice



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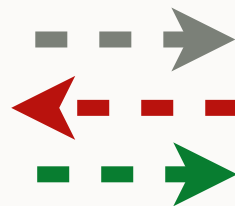
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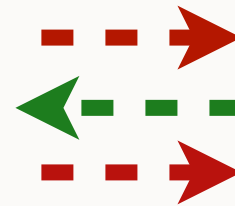
Charlene



Alice



Intruder



Unknown



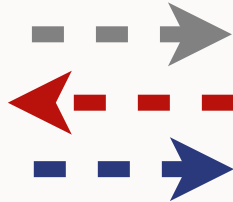
Bob

# CONTEXT

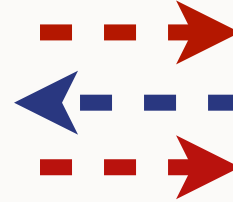
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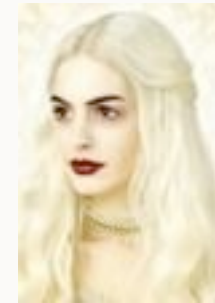
Alice



Intruder



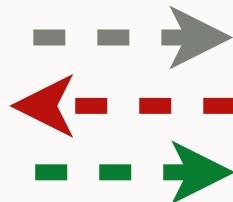
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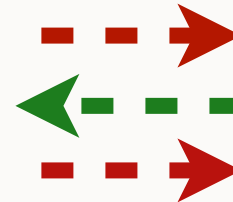
Charlene



Alice



Intruder



Unknown



Bob

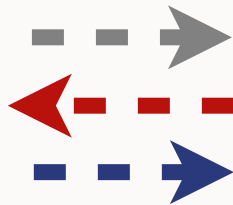
Can the intruder distinguish the two situations ?

# CONTEXT

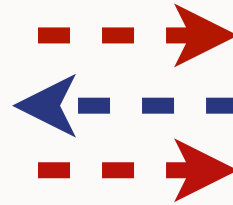
- Equivalence properties : strong secret, anonymity,...



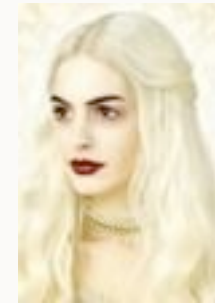
Alice



Intruder



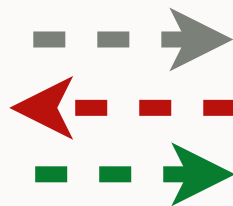
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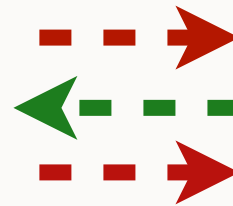
Charlene



Alice



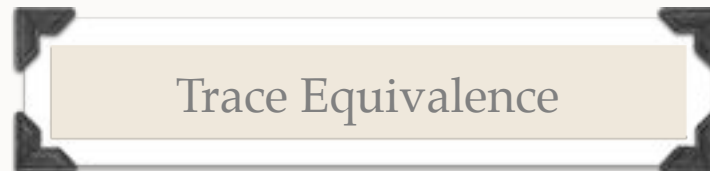
Intruder



Unknown



Bob

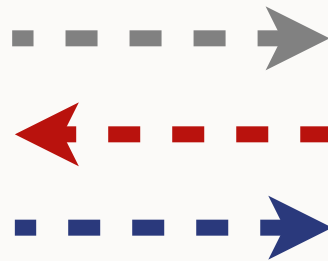


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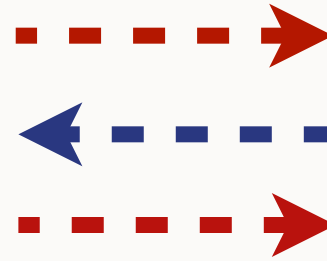
- Knowledge indistinguishability : static equivalence



Alice



Intruder



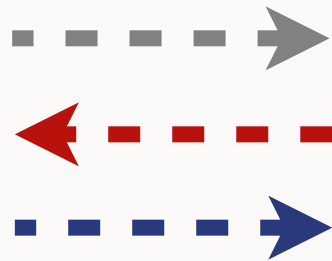
Bob

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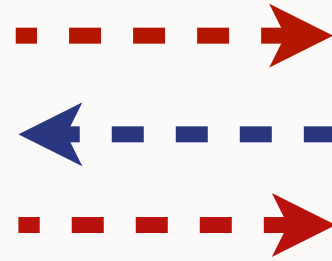
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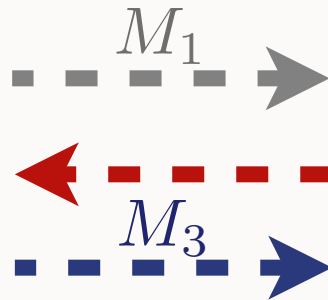
Bob

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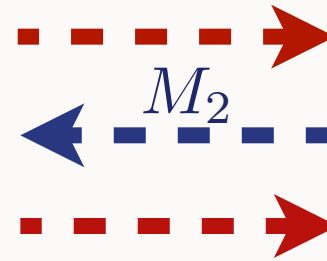
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Alice



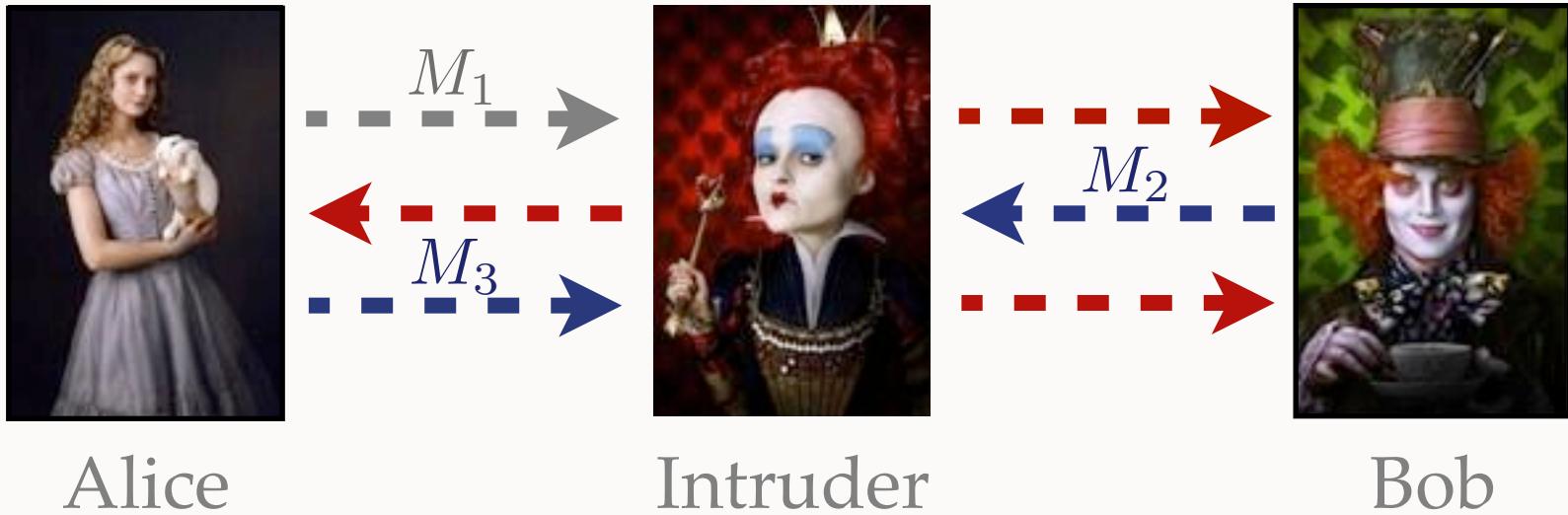
Intruder



Bob

# CONTEXT

- Knowledge indistinguishability : static equivalence

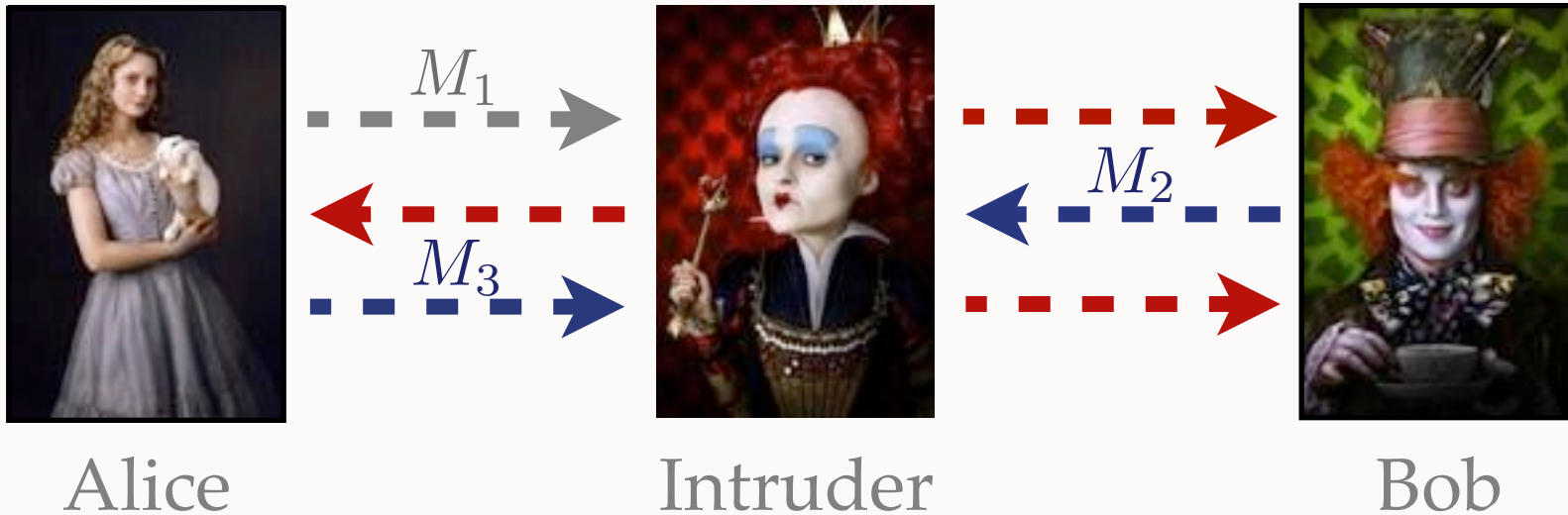


Example with decryption :  $dec(\{x\}_y, y) = x$



# CONTEXT

- Knowledge indistinguishability : static equivalence



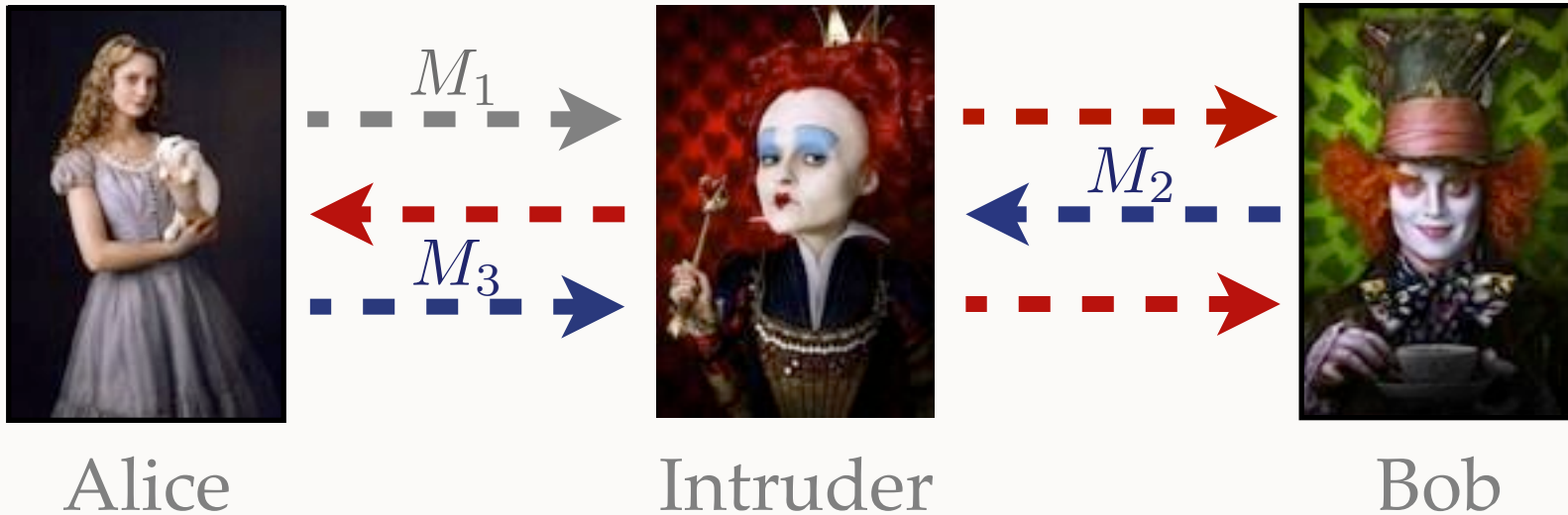
Example with decryption :  $dec(\{x\}_y, y) = x$

$$\Phi_1 : a, \{b\}_a, b$$

$$\Phi_2 : c, \{b\}_a, b$$

# CONTEXT

- Knowledge indistinguishability : static equivalence



Example with decryption :  $dec(\{x\}_y, y) = x$

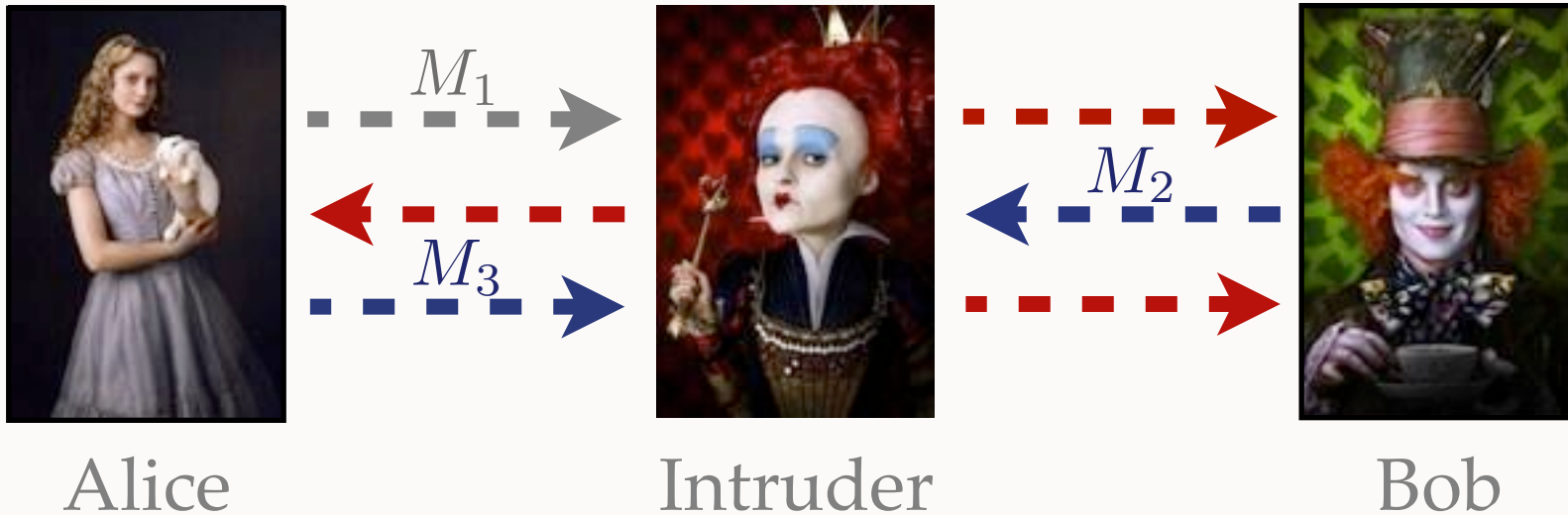
$$dec(M_2, M_1) = M_3$$

$$\Phi_1 : a, \{b\}_a, b$$

$$\Phi_2 : c, \{b\}_a, b$$

# CONTEXT

- Knowledge indistinguishability : static equivalence



Example with decryption :  $dec(\{x\}_y, y) = x$

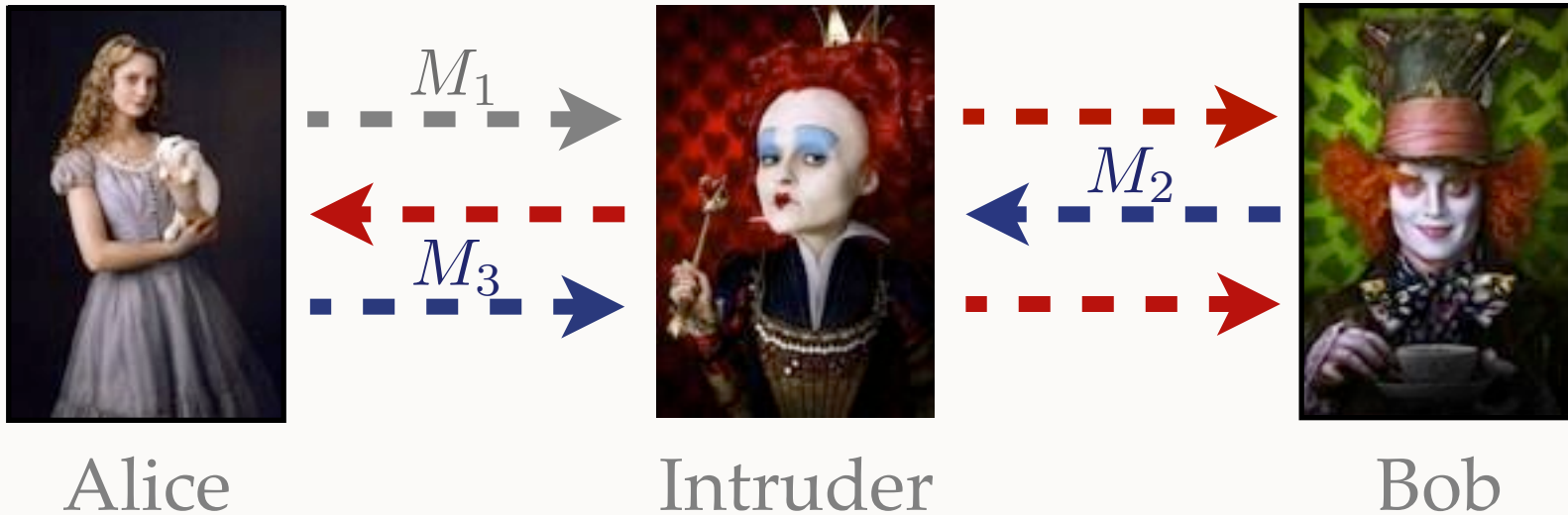
$$dec(M_2, M_1) = M_3$$

$$\Phi_1 : a, \{b\}_a, b \quad dec(\{b\}_a, a) = b$$

$$\Phi_2 : c, \{b\}_a, b \quad dec(\{b\}_a, c) \neq b$$

# CONTEXT

- Knowledge indistinguishability : static equivalence



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$$dec(M_2, M_1) = M_3$$

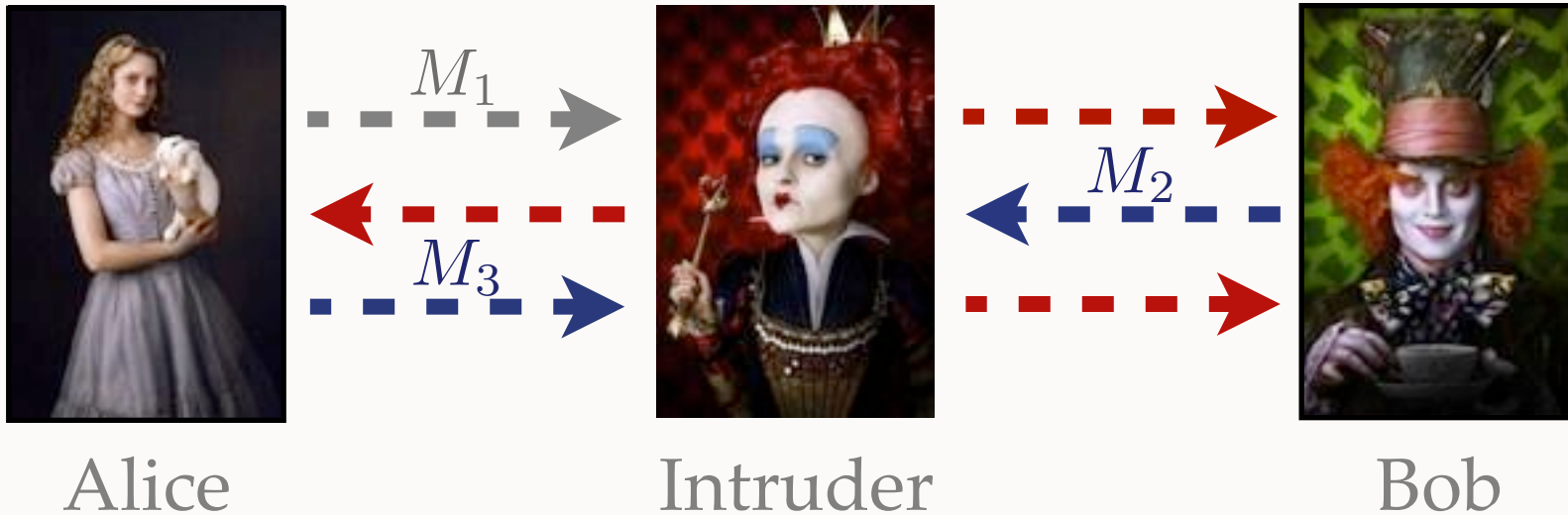
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Not equivalent

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- Knowledge indistinguishability : static equivalence



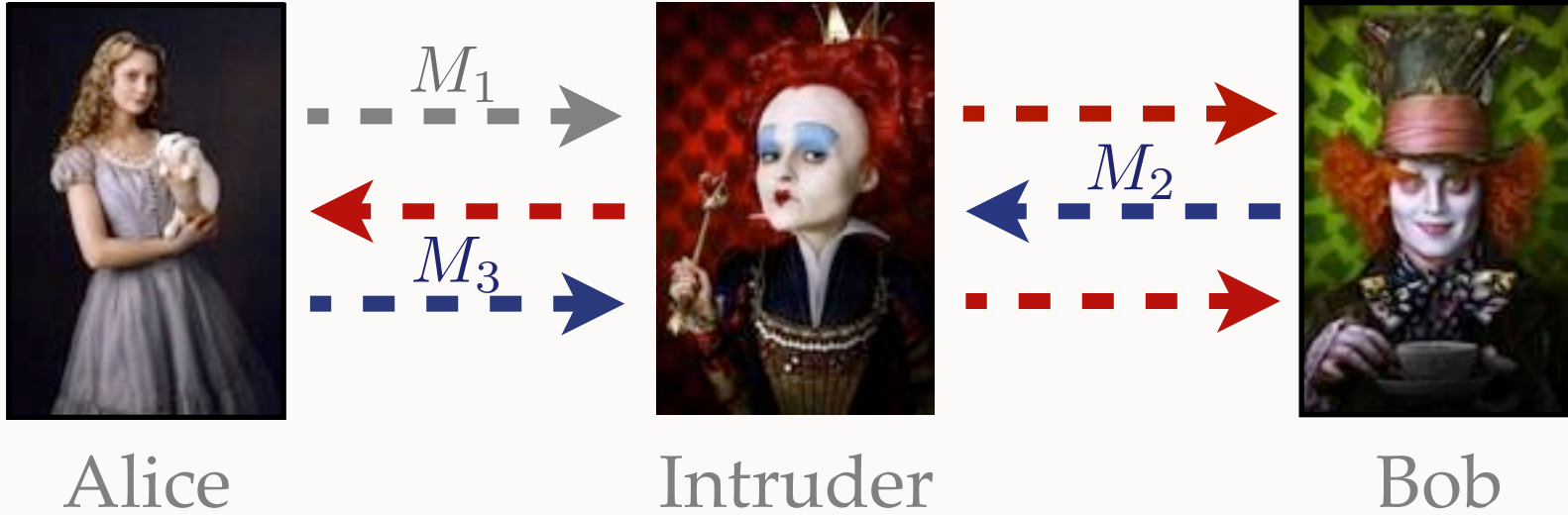
Example with decryption :  $dec(\{x\}_y, y) = x$

	$dec(M_2, M_1) = M_3$	
$\Phi_1 : a, \{b\}_a, b$	$dec(\{b\}_a, a) = b$	$\Phi_1 : a, \{b\}_a$
$\Phi_2 : c, \{b\}_a, b$	$dec(\{b\}_a, c) \neq b$	$\Phi_2 : c, \{b\}_a$

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- Knowledge indistinguishability : static equivalence



Example with decryption :  $dec(\{x\}_y, y) = x$

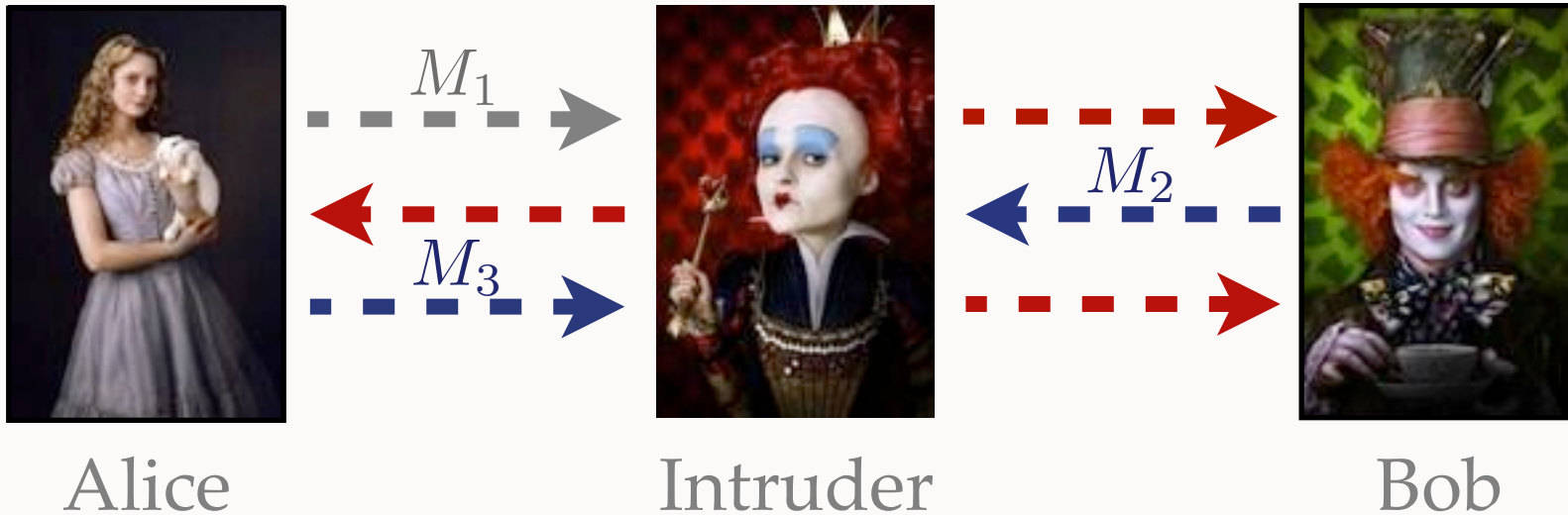
	$dec(M_2, M_1) = M_3$		$\Phi_1 : a, \{b\}_a$
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Not equivalent

No test

# CONTEXT

- Knowledge indistinguishability : static equivalence



Example with decryption :  $dec(\{x\}_y, y) = x$

	$dec(M_2, M_1) = M_3$	
$\Phi_1 : a, \{b\}_a, b$	$dec(\{b\}_a, a) = b$	
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Not equivalent

$\Phi_1 : a, \{b\}_a$		No test
$\Phi_2 : c, \{b\}_a$		

Equivalent

# PREVIOUS WORKS

Most of the previous works focus on stronger equivalence

- A. Tiu and J. E. Dawson. *Automating open bisimulation checking for the spi calculus.*
- M. Baudet. *Sécurité des protocoles cryptographiques : aspects logiques et calculatoires.* Phd thesis
- B. Blanchet, M. Abadi, and C. Fournet. *Automated verification of selected equivalences for security protocols.*

→ Tool : B. Blanchet, *ProVerif*

Trace equivalence for simple processes without else branches

- V. Cortier and S. Delaune. *A method for proving observational equivalence.*



# MOTIVATION

## ■ Example

Two problematic examples :

- e-passport protocols : M. Arapinis, T. Chothia, E. Ritter, and M. Ryan. *Analysing unlinkability and anonymity using the applied pi calculus.*
- private authentication protocol : M. Abadi and C. Fournet. *Private authentication. Theoretical Computer Science.*

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Alice

$\{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)}$



Bob

# MOTIVATION

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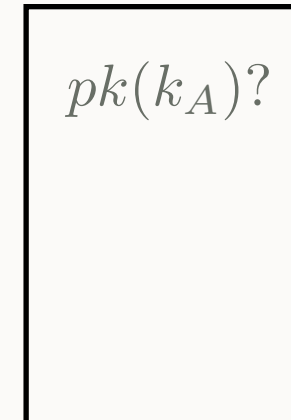

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# MOTIVATION

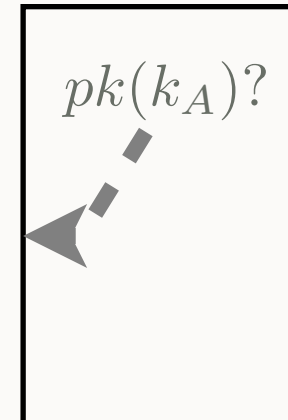
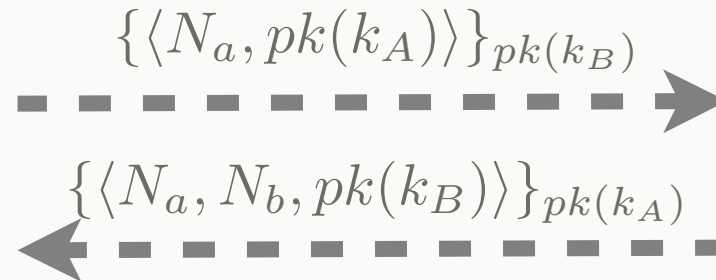
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# MOTIVATION

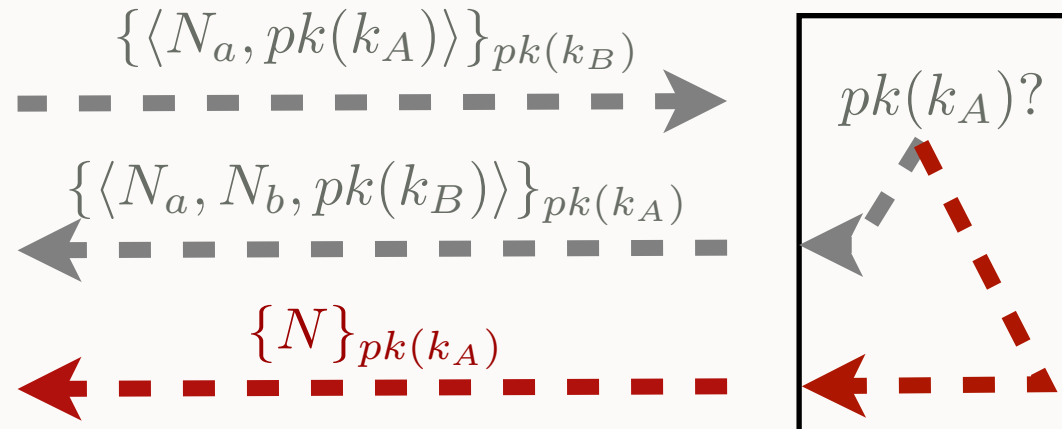
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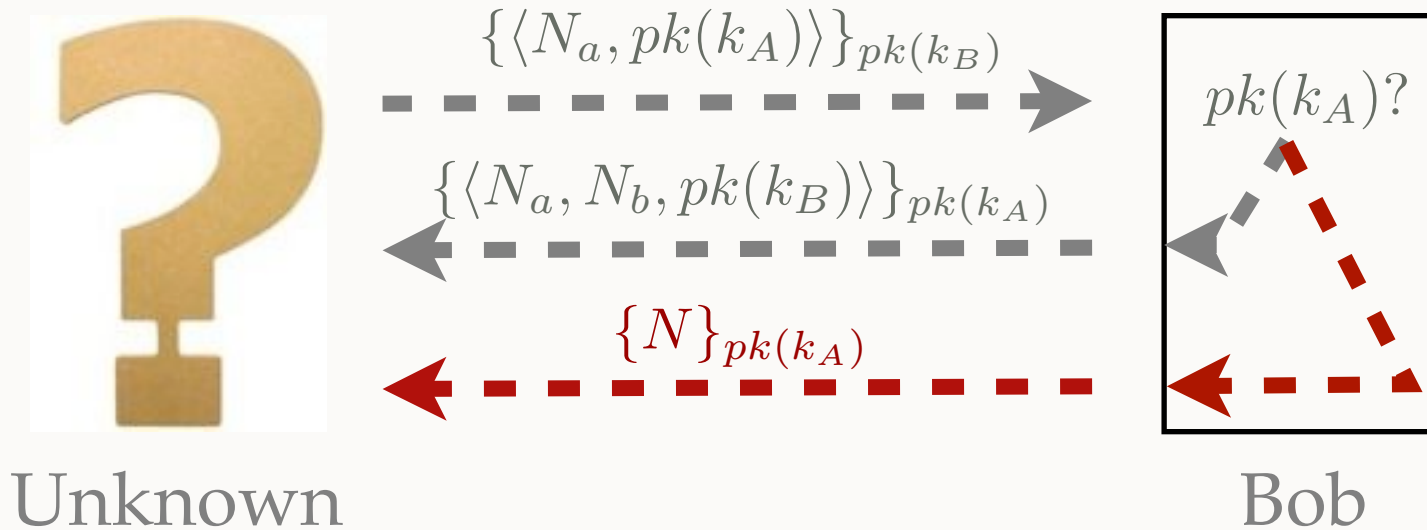
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# MOTIVATION

- Example



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Intruder



Bob



Charlene



Intruder



Bob

# MOTIVATION

- Example



Alice



Bob



Charlene



Bob

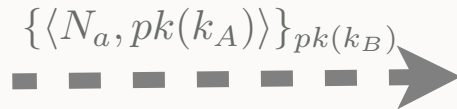


# MOTIVATION

- Example



Alice



Bob



Charlene



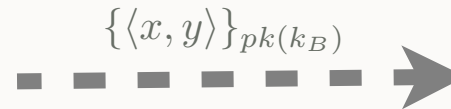
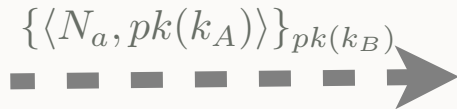
Bob

# MOTIVATION

- Example



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Charlene



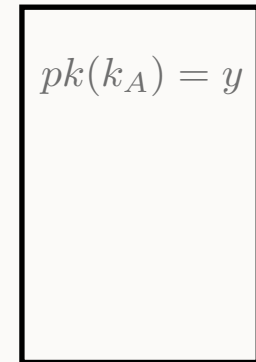
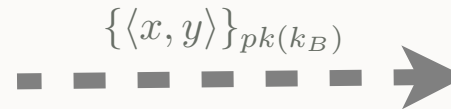
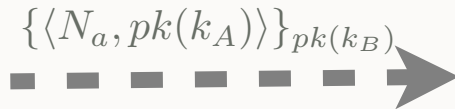
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# MOTIVATION

- Example



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Charlene



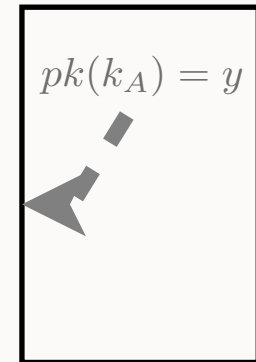
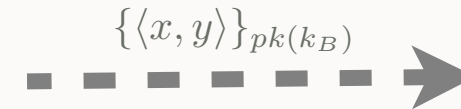
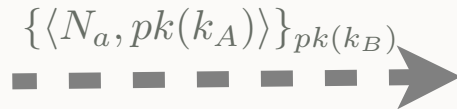
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# MOTIVATION

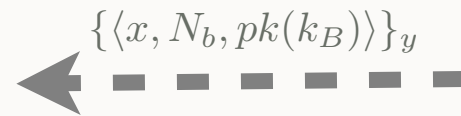
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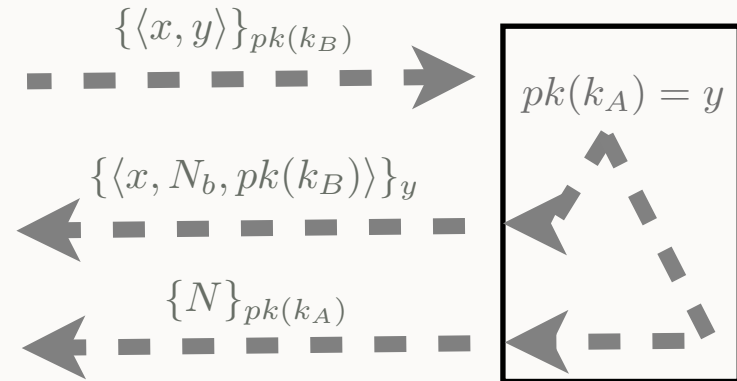
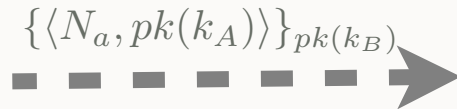
## ■ Example



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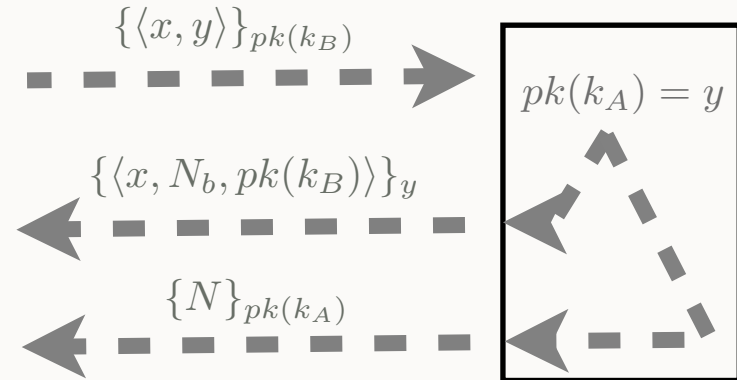
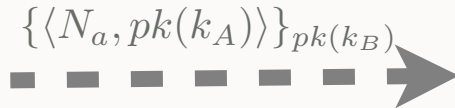
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# MOTIVATION

## ■ Example



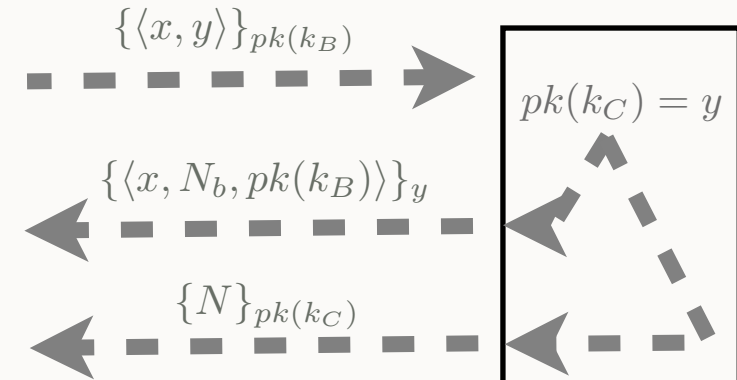
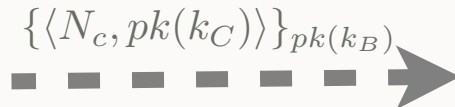
Alice



Bob



Charlene



Bob

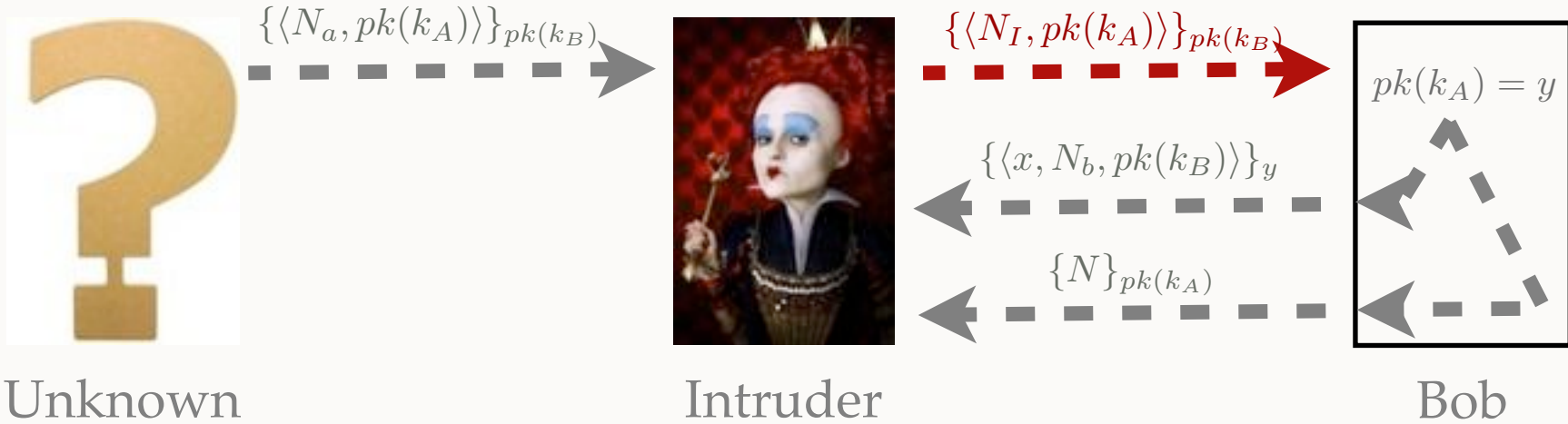
# MOTIVATION

## ■ Example



# MOTIVATION

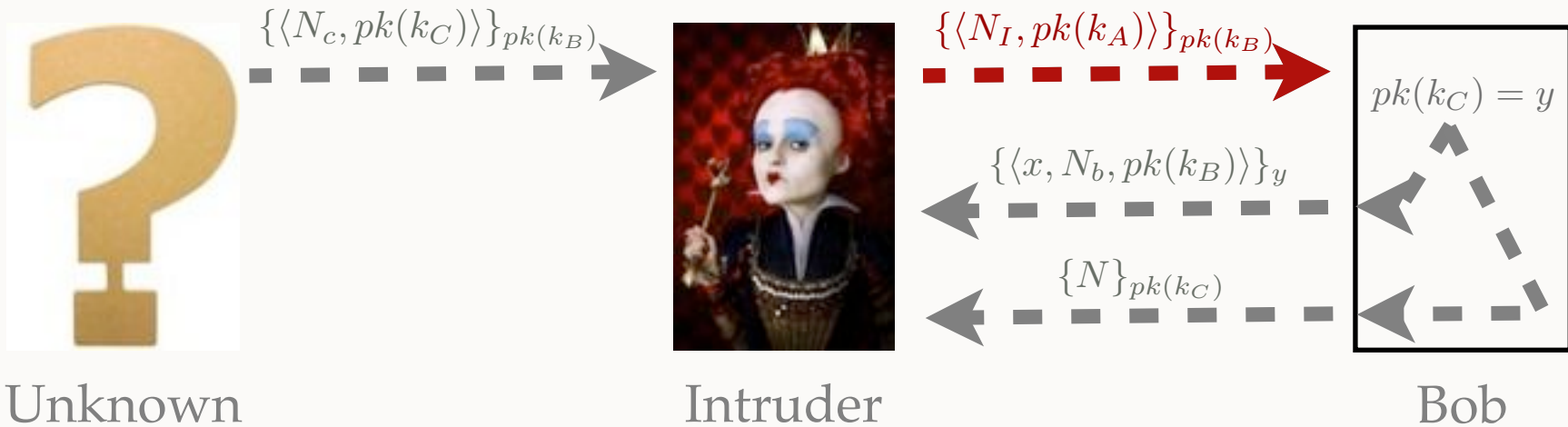
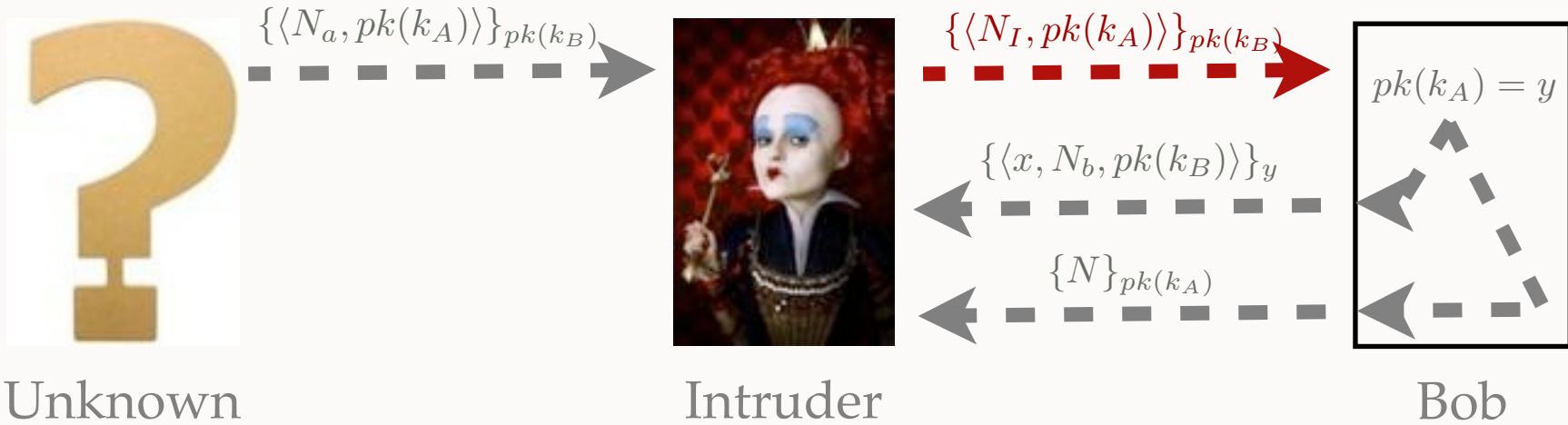
## ■ Example





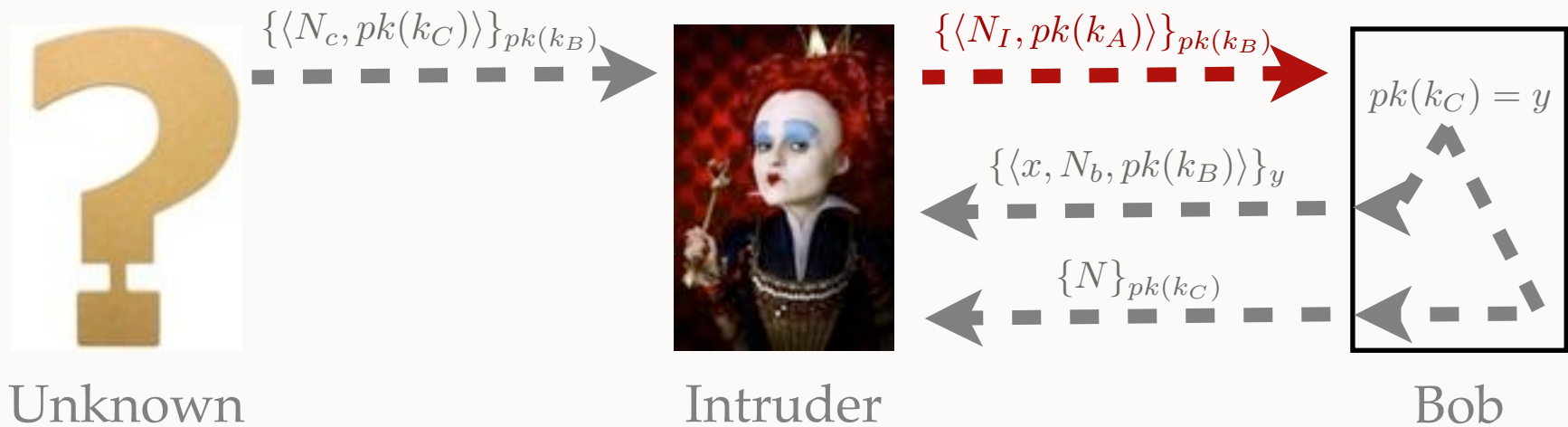
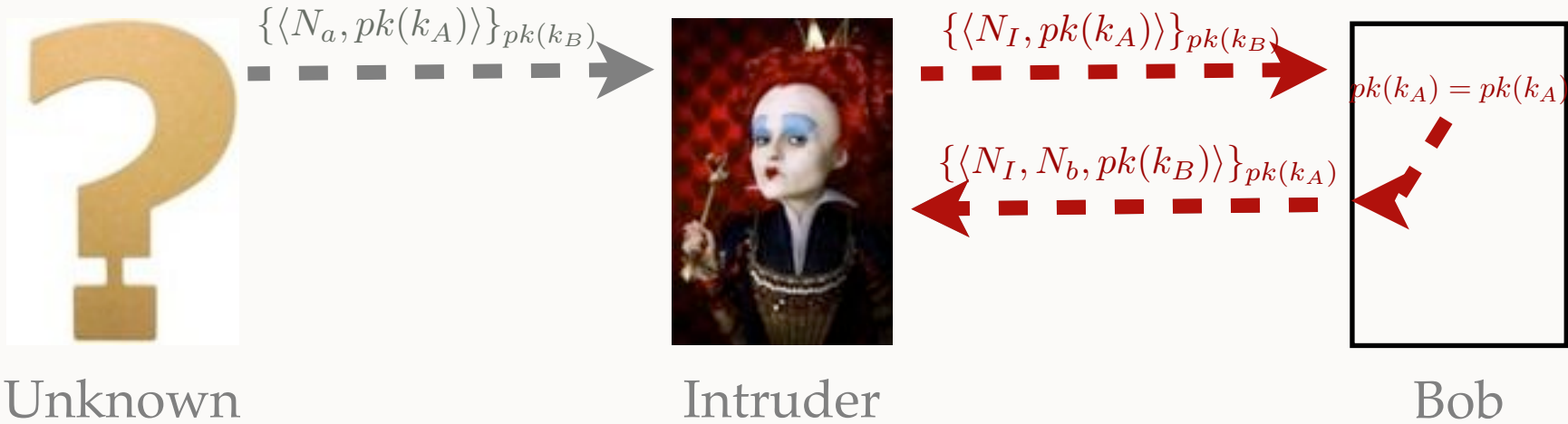
# MOTIVATION

## ■ Example



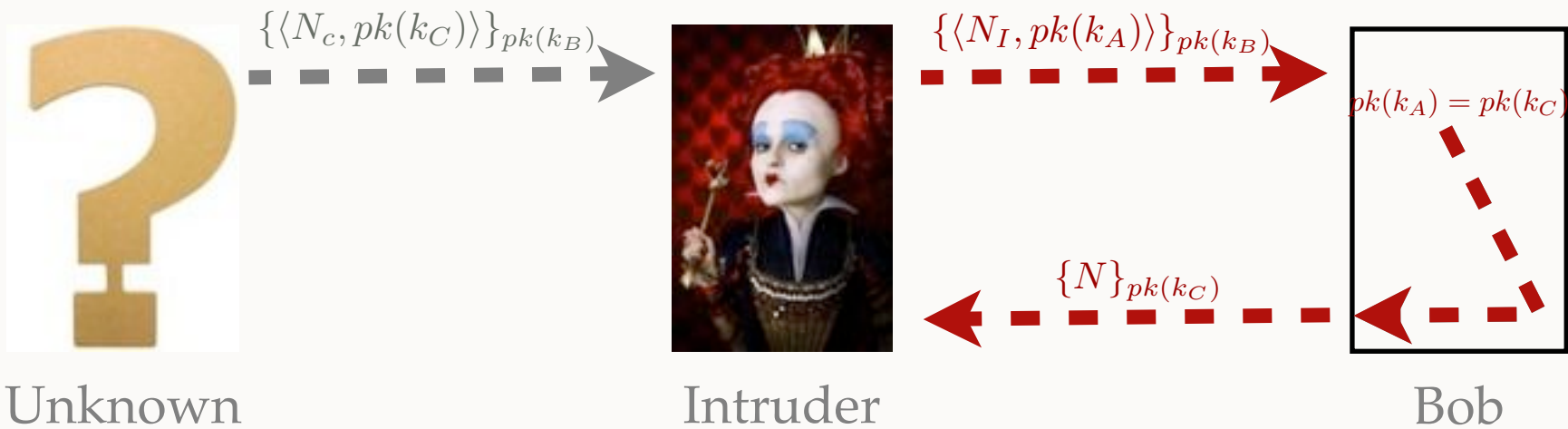
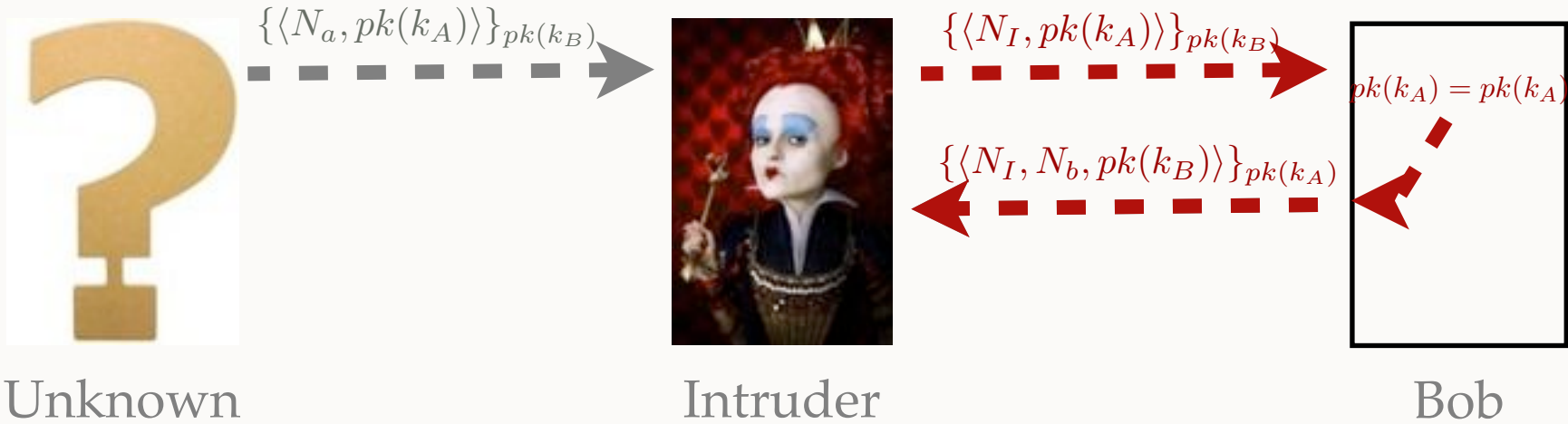
# MOTIVATION

## ■ Example



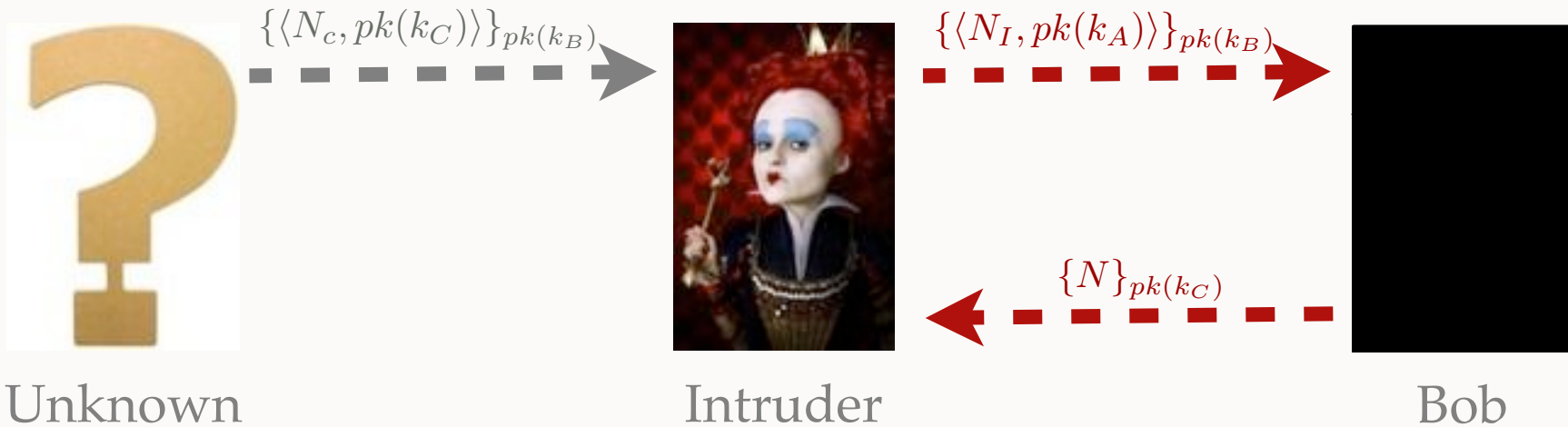
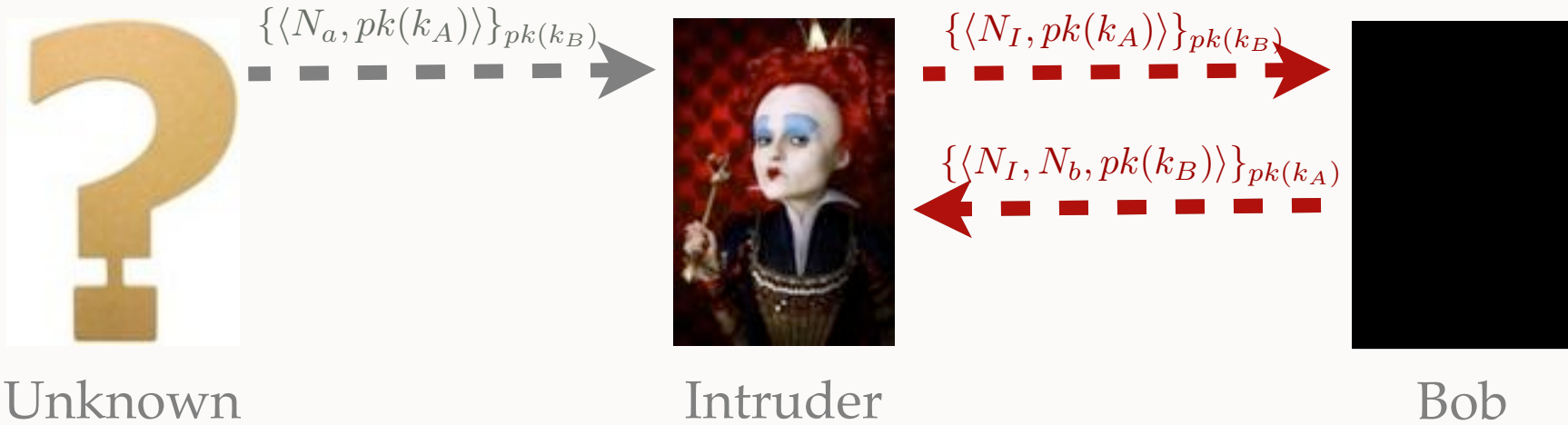
# MOTIVATION

## ■ Example



# MOTIVATION

## ■ Example



# CONTRIBUTION

## Decision procedure for verification of trace equivalence

- Infinitely many traces are represented by symbolic constraint system
- + Protocol possibly non-determinist and with non trivial else branches
- + Private channels
- Fixed set of cryptographic primitives : symmetric and asymmetric encryption, pairing and signature
- Bounded number of sessions (no replication in the process algebra)

# CONSTRAINT SYSTEM

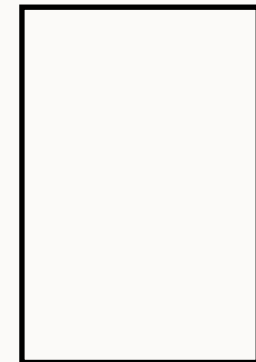
- One constraint system = one interleaving = several traces



Alice



Intruder



Bob

# CONSTRAINT SYSTEM

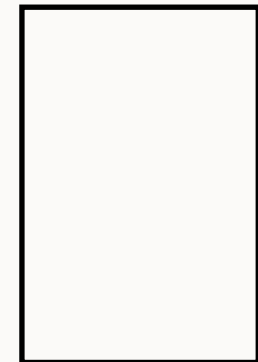
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Alice



Intruder



Bob

$pk(k_A), pk(k_B), pk(k_C), N_I$

# CONSTRAINT SYSTEM

- One constraint system = one interleaving = several traces

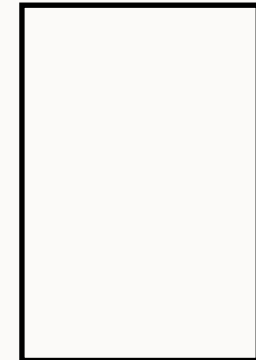


Alice

$\{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)}$   
----->



Intruder



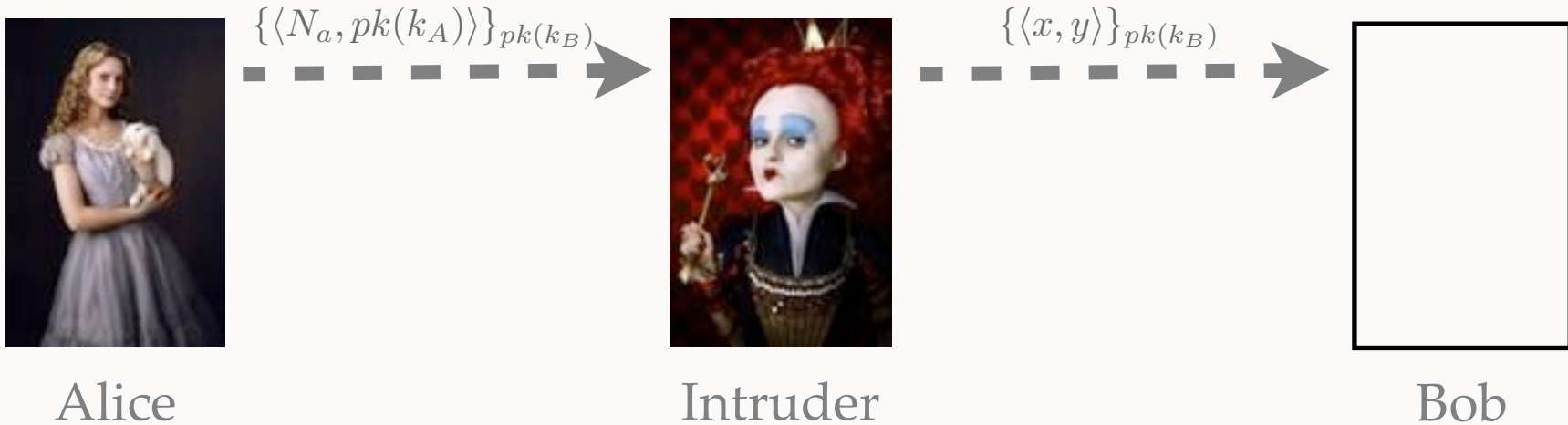
Bob

$pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)}$



# CONSTRAINT SYSTEM

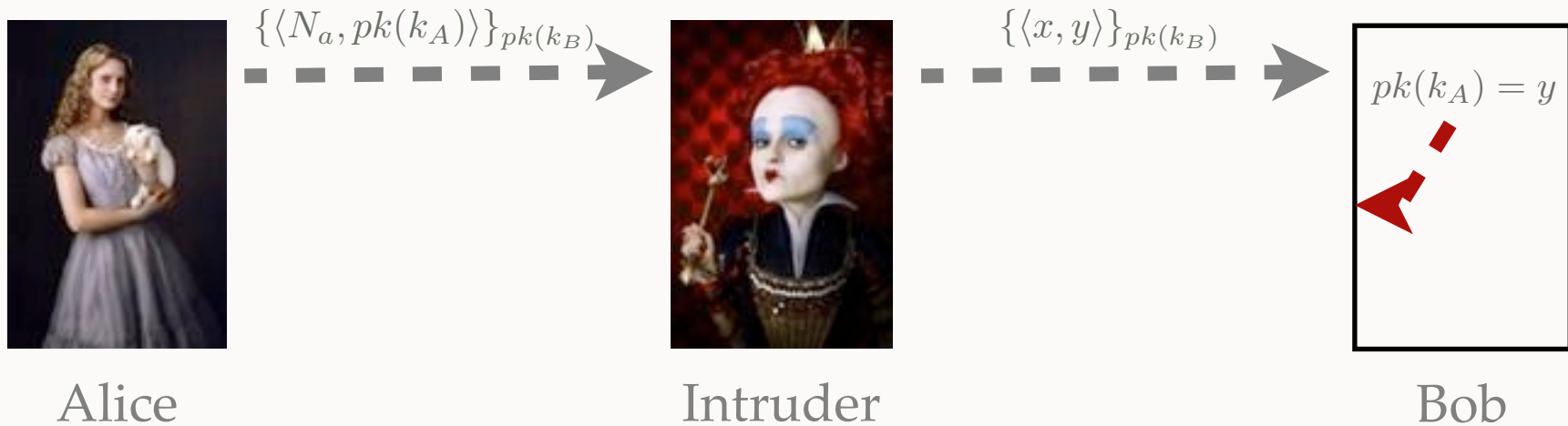
- One constraint system = one interleaving = several traces



$$pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)} \stackrel{?}{\vdash} \{\langle x, y \rangle\}_{pk(k_B)}$$

# CONSTRAINT SYSTEM

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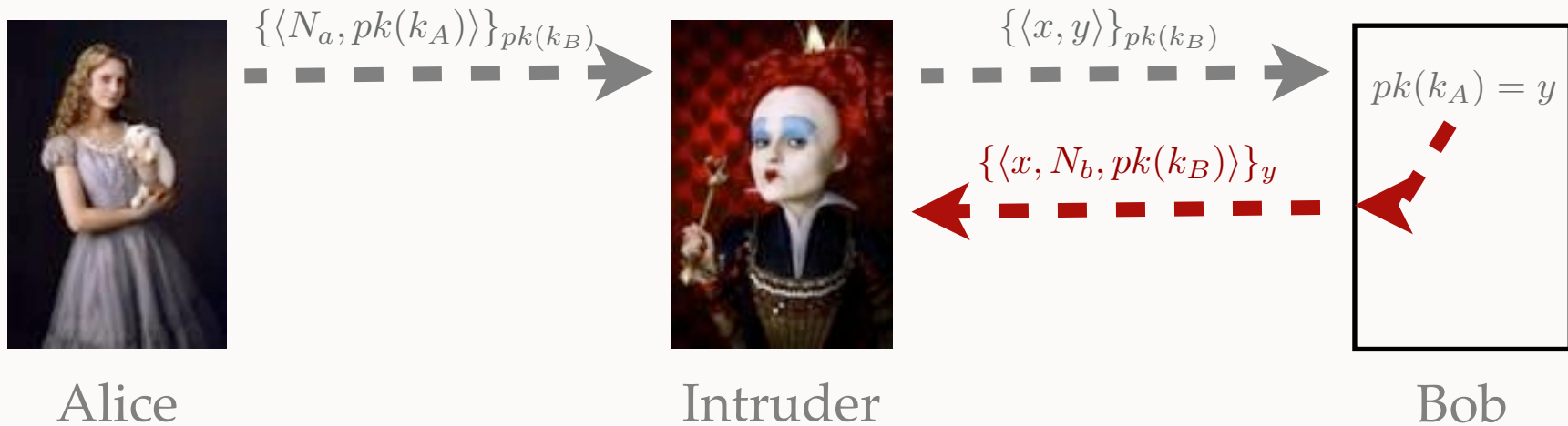


$$pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)} \stackrel{?}{\vdash} \{\langle x, y \rangle\}_{pk(k_B)}$$

$$y \stackrel{?}{=} pk(k_A)$$

# CONSTRAINT SYSTEM

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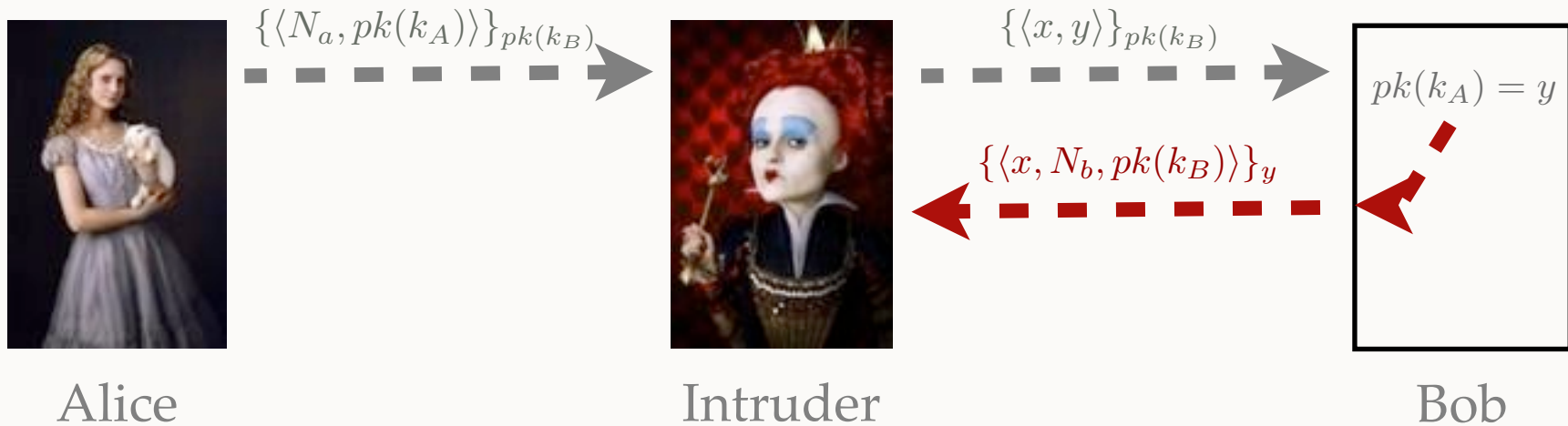
$$pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)} \stackrel{?}{\vdash} \{\langle x, y \rangle\}_{pk(k_B)}$$

$$pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)}, \{\langle x, N_b, pk(k_B) \rangle\}_y$$

$$y \stackrel{?}{=} pk(k_A)$$

# CONSTRAINT SYSTEM

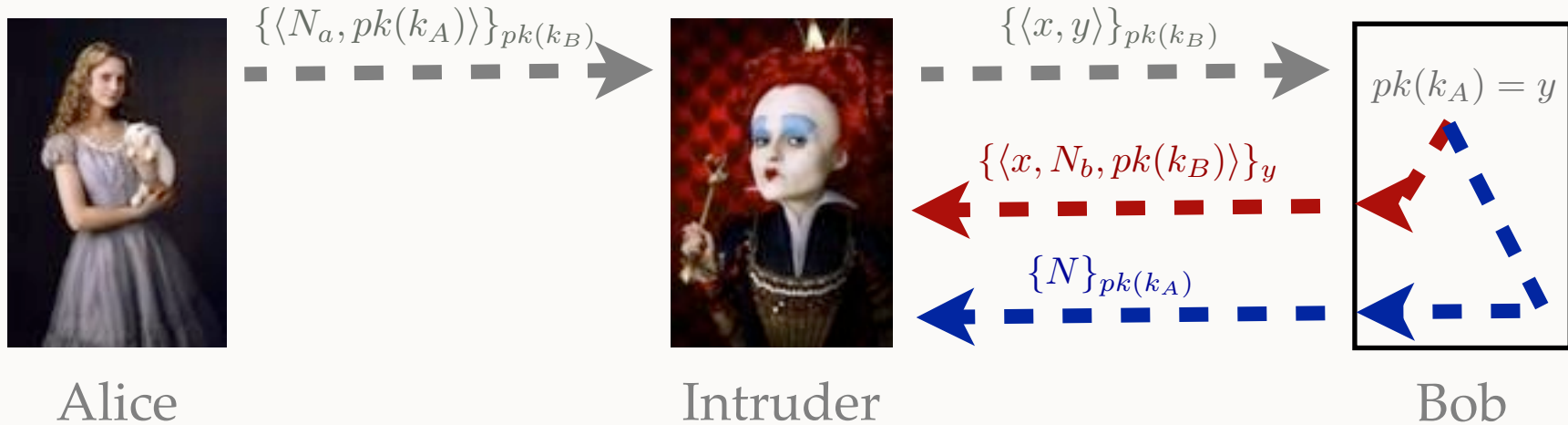
- One constraint system = one interleaving = several traces



$$\begin{aligned}
 D &: pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)} \stackrel{?}{\vdash} \{\langle x, y \rangle\}_{pk(k_B)} \\
 \Phi &: pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)}, \{\langle x, N_b, pk(k_B) \rangle\}_y \\
 E &: y \stackrel{?}{=} pk(k_A)
 \end{aligned}$$

# CONSTRAINT SYSTEM

- One constraint system = one interleaving = several traces



$$D : pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)} \stackrel{?}{\vdash} \{\langle x, y \rangle\}_{pk(k_B)}$$

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$$\Phi : pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)}, \{N\}_{pk(k_A)}$$

$$E : y \neq pk(k_A)$$

# CONSTRAINT SYSTEM

- One solution of a constraint system = one trace

$D : pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)} \vdash \{\langle x, y \rangle\}_{pk(k_B)}$

$\Phi : pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)}, \{\langle x, N_b, pk(k_B) \rangle\}_y$

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A solution is a pair of substitution  $(\sigma, \theta)$  where :

- $\sigma$  describe the messages
- $\theta$  describe how the messages are deduced

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$\sigma = \{x \rightarrow N_I; y \rightarrow pk(k_A)\}$



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$$\Phi : pk(k_A), pk(k_B), pk(k_C), N_I, \{\langle N_a, pk(k_A) \rangle\}_{pk(k_B)}, \{\langle x, N_b, pk(k_B) \rangle\}_y$$

$$ax_1 \quad ax_2 \quad ax_3 \quad ax_4 \quad ax_5 \quad ax_6$$

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$$\theta = \{X_1 \rightarrow \{\langle ax_4, ax_1 \rangle\}_{ax_2}\}$$

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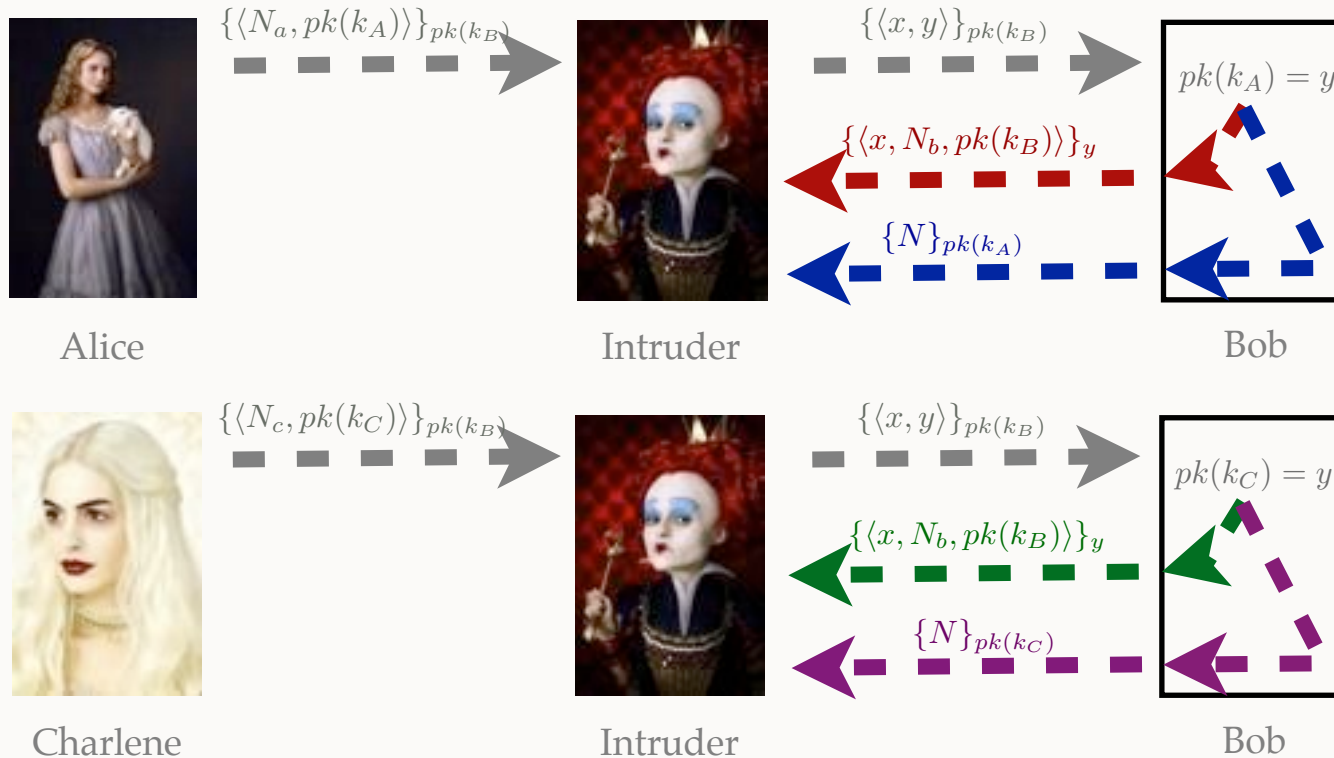
$$\sigma = \{x \rightarrow N_a; y \rightarrow pk(k_A)\}$$

$$\theta = \{X_1 \rightarrow \{\langle ax_4, ax_1 \rangle\}_{ax_2}\}$$

$$\theta = \{X_1 \rightarrow ax_5\}$$

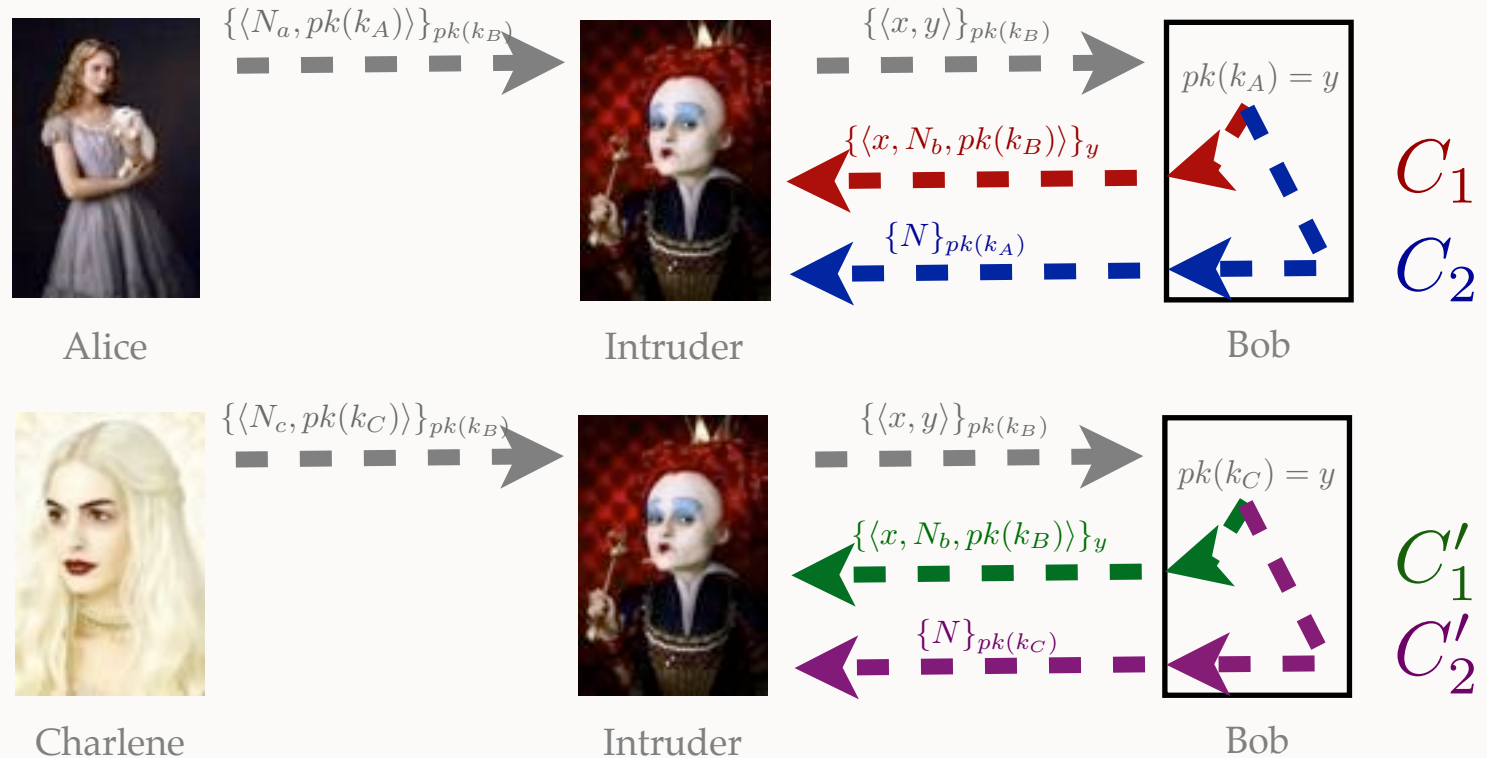
# CONSTRAINT SYSTEM

- Set of constraint systems



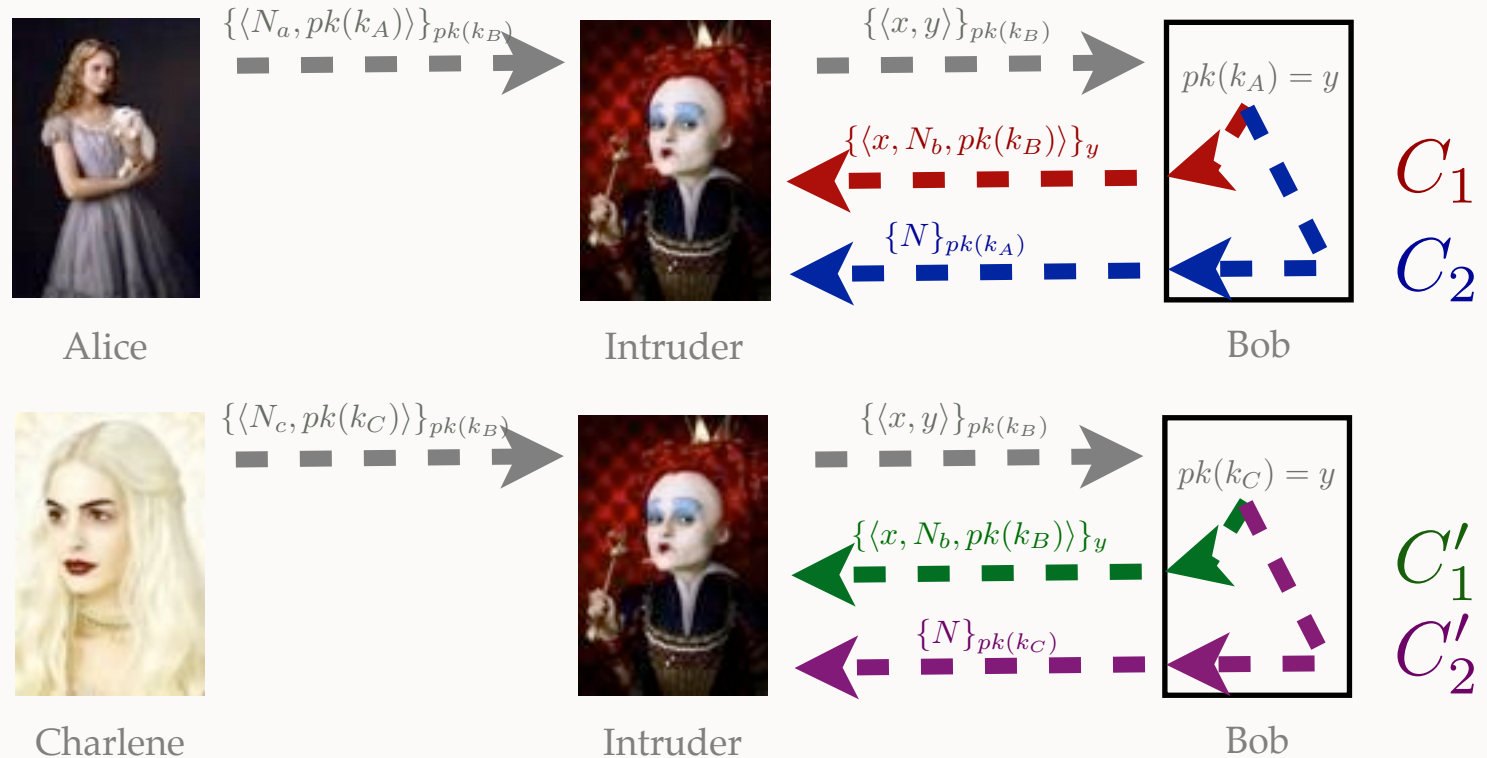
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# CONSTRAINT SYSTEM

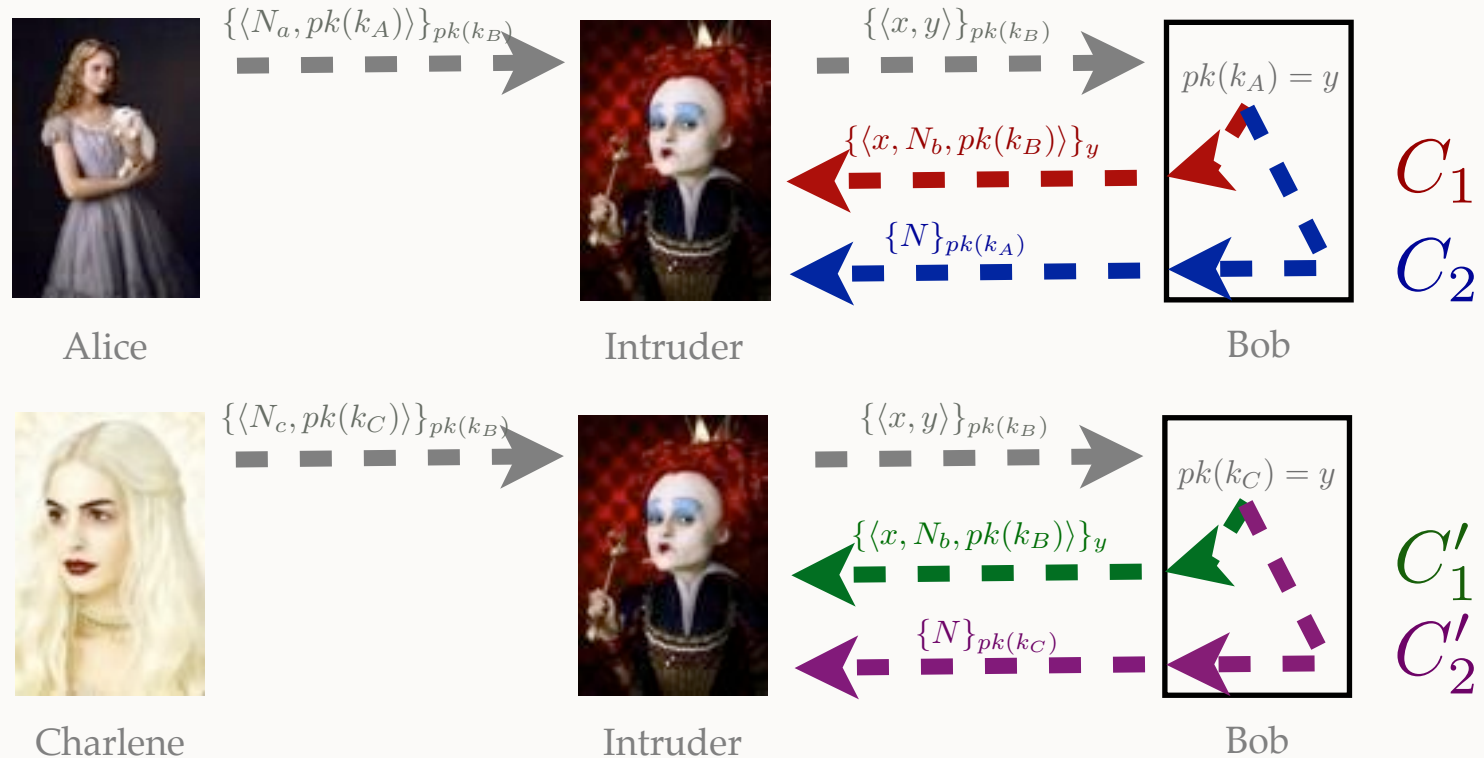
- Set of constraint systems



$$\{C_1; C_2\} \approx \{C'_1; C'_2\}$$

# CONSTRAINT SYSTEM

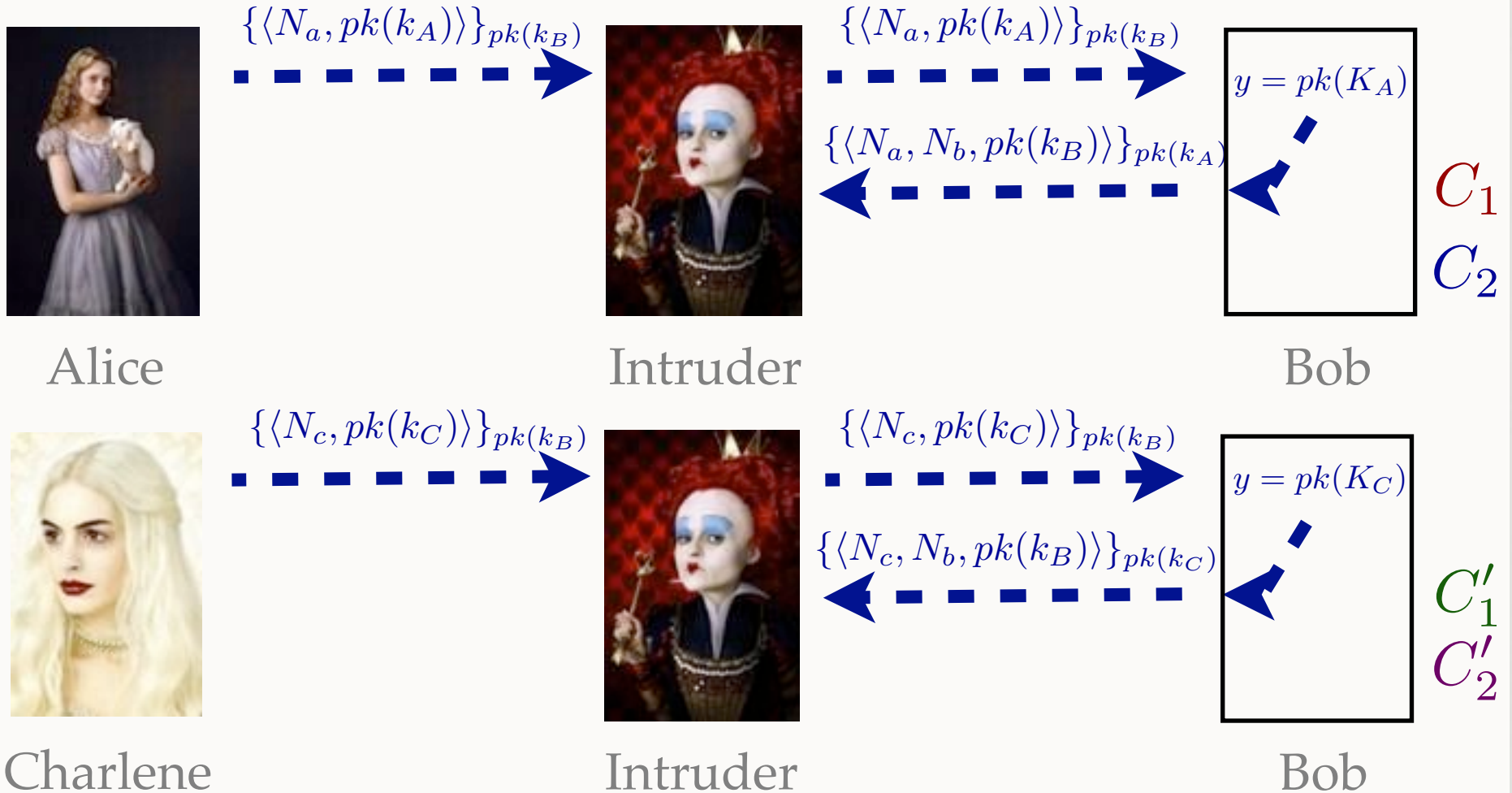
- Set of constraint systems



Symbolic equivalence between sets of constraint systems

# CONSTRAINT SYSTEM

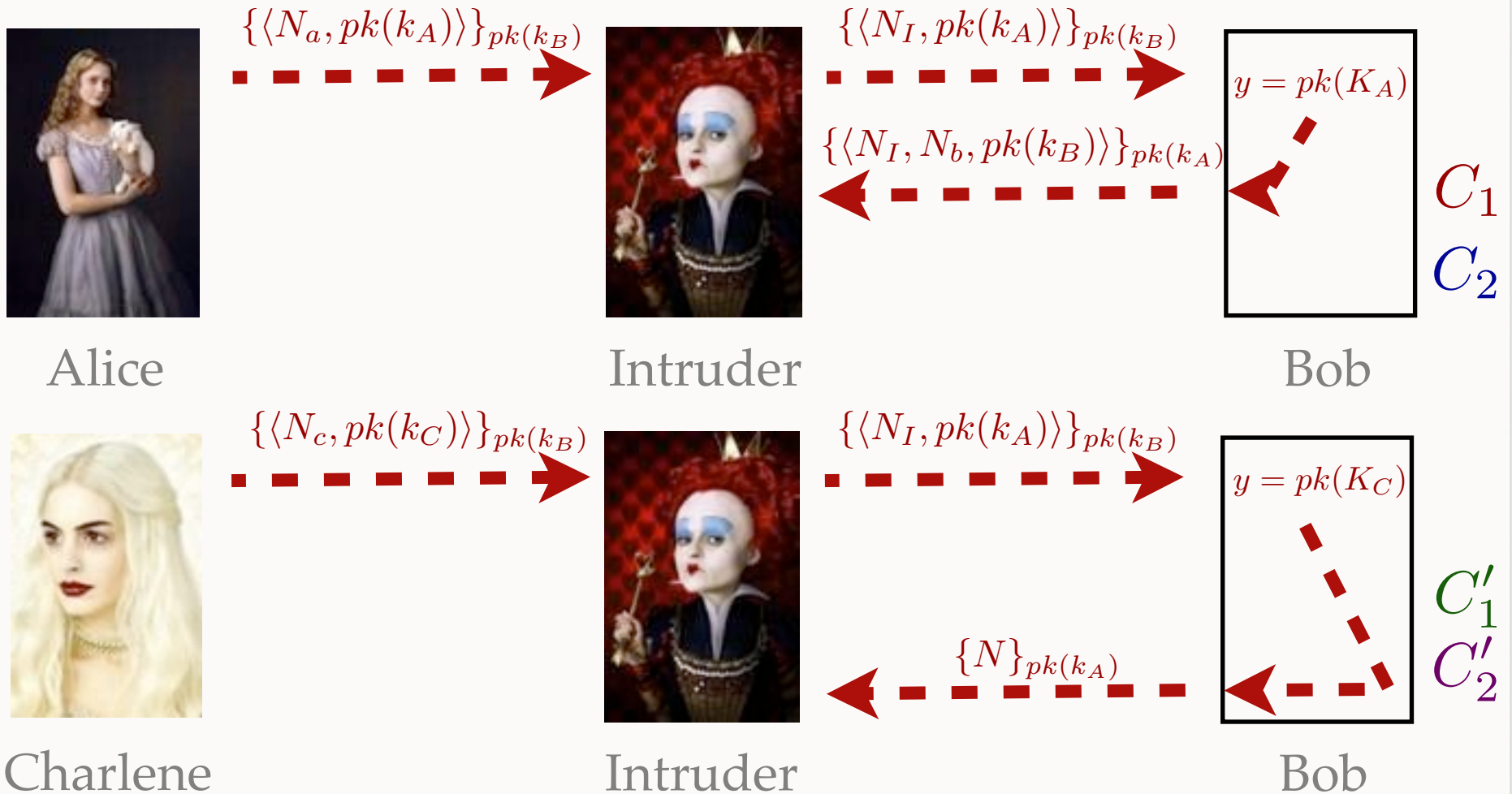
- Why sets of constraint systems are necessary ?





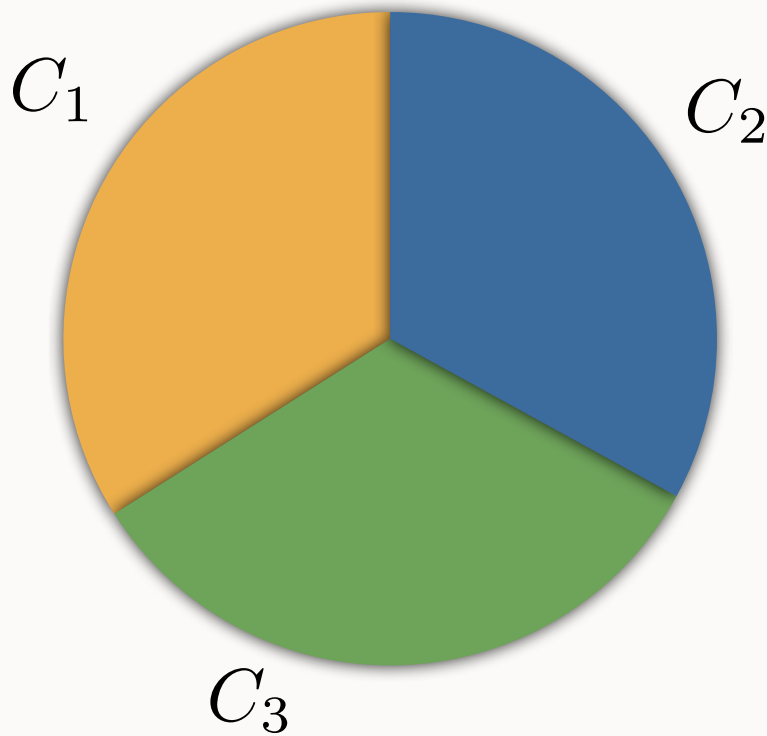
# CONSTRAINT SYSTEM

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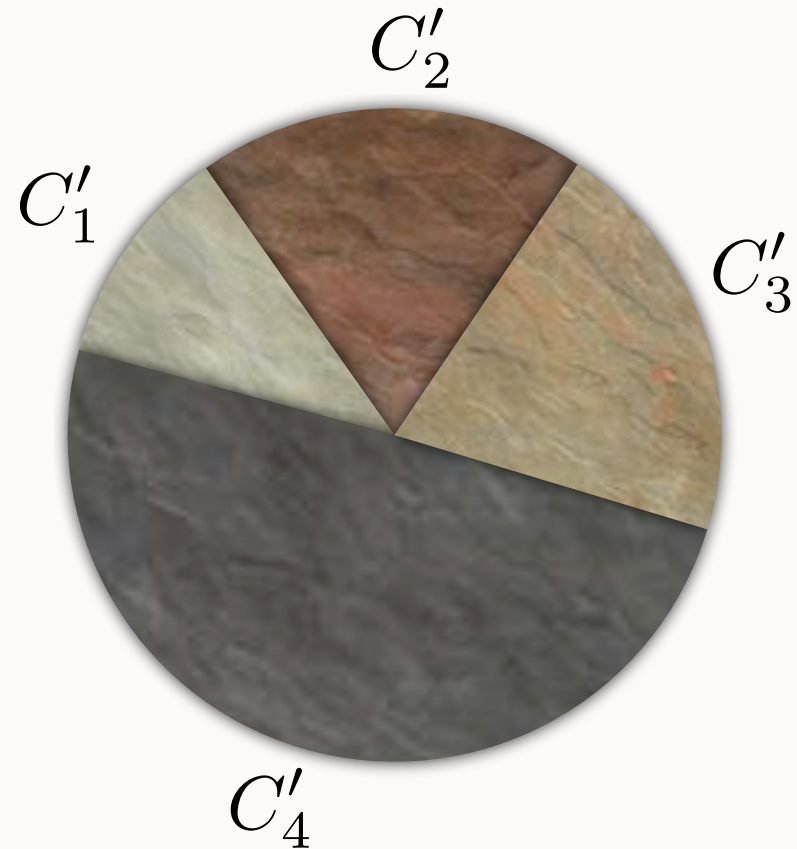


# CONSTRAINT SYSTEM

- Why sets of constraint systems are necessary ?



$$S = \{C_1; C_2; C_3\}$$



$$S' = \{C'_1; C'_2; C'_3; C'_4\}$$

# CONSTRAINT SYSTEM

- Symbolic equivalence between sets of constraint systems

To check whether  $P$  and  $P'$  are trace equivalent, we have to check that :

$$S \approx S', \text{ for all symbolic interleaving}$$

## Symbolic equivalence $S \approx S'$

- For all  $C \in S$ , for all  $(\theta, \sigma) \in \text{Sol}(C)$ , there exists  $C' \in S'$  and  $\sigma'$  such that  $(\theta, \sigma') \in \text{Sol}(C')$  and  $\Phi\sigma \sim \Phi'\sigma'$
- and conversely

# CONSTRAINT SYSTEM

## ■ Previous works on constraint system

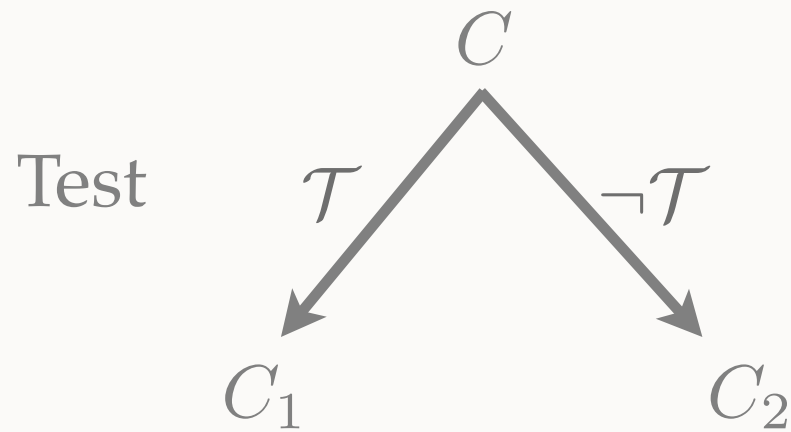
1. M. Baudet. *Sécurité des protocoles cryptographiques : aspects logiques et calculatoires*. Phd thesis
2. Y. Chevalier and M. Rusinowitch. *Decidability of equivalence of symbolic derivations*.
3. V. Cortier and S. Delaune. *A method for proving observational equivalence*.
4. A. Tiu and J. E. Dawson. *Automating open bisimulation checking for the spi calculus*.
5. V. Cheval, H. Comon-Lundh, S. Delaune. *Automating security analysis: symbolic equivalence of constraint systems*

### **Focus on :**

- symbolic equivalence between two constraint systems (All)
- positive constraint system (no disequations) (All)
- subterm convergent equational theory (1,2 & 3)
- more restricted equational theory (4 & 5)

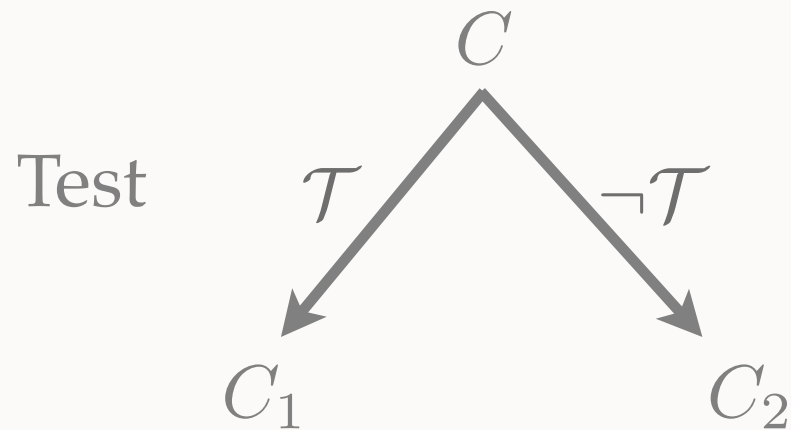
# THE ALGORITHM

- Set of rules



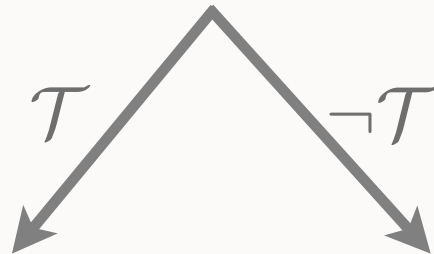
# THE ALGORITHM

- Set of rules



- How to apply the rules :

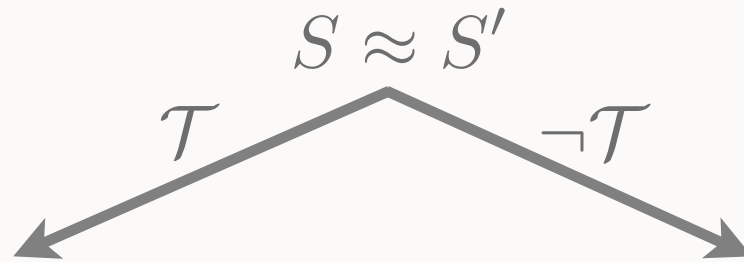
$$\{C^1; C^2; \dots\} \approx \{C^n; \dots\}$$



$$\{C_1^1; C_1^2; \dots\} \approx \{C_1^n; \dots\} \quad \{C_2^1; C_2^2; \dots\} \approx \{C_2^n; \dots\}$$

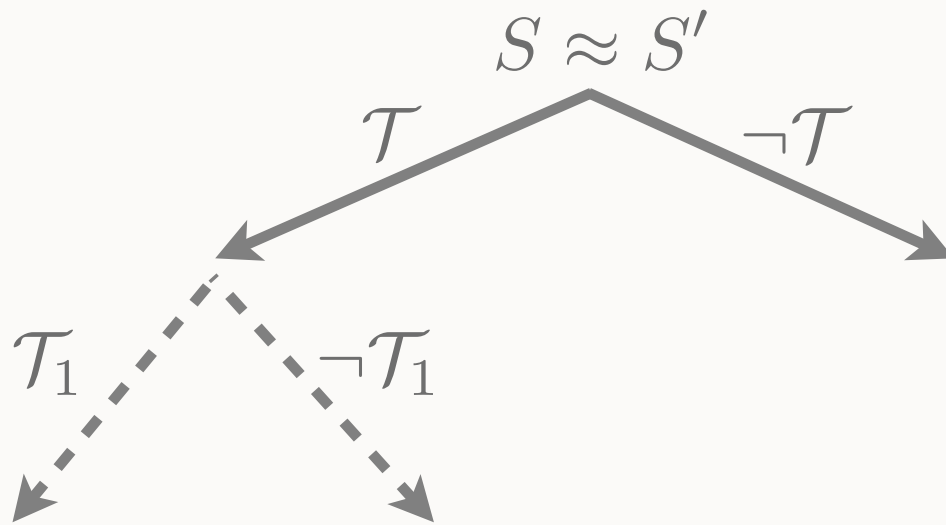
# THE ALGORITHM

- A complete execution



# THE ALGORITHM

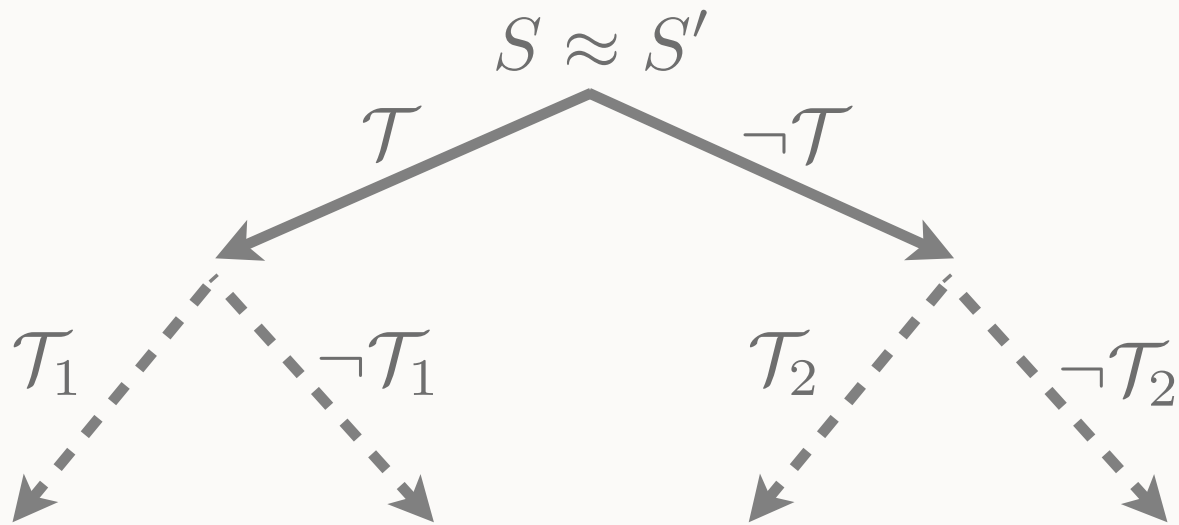
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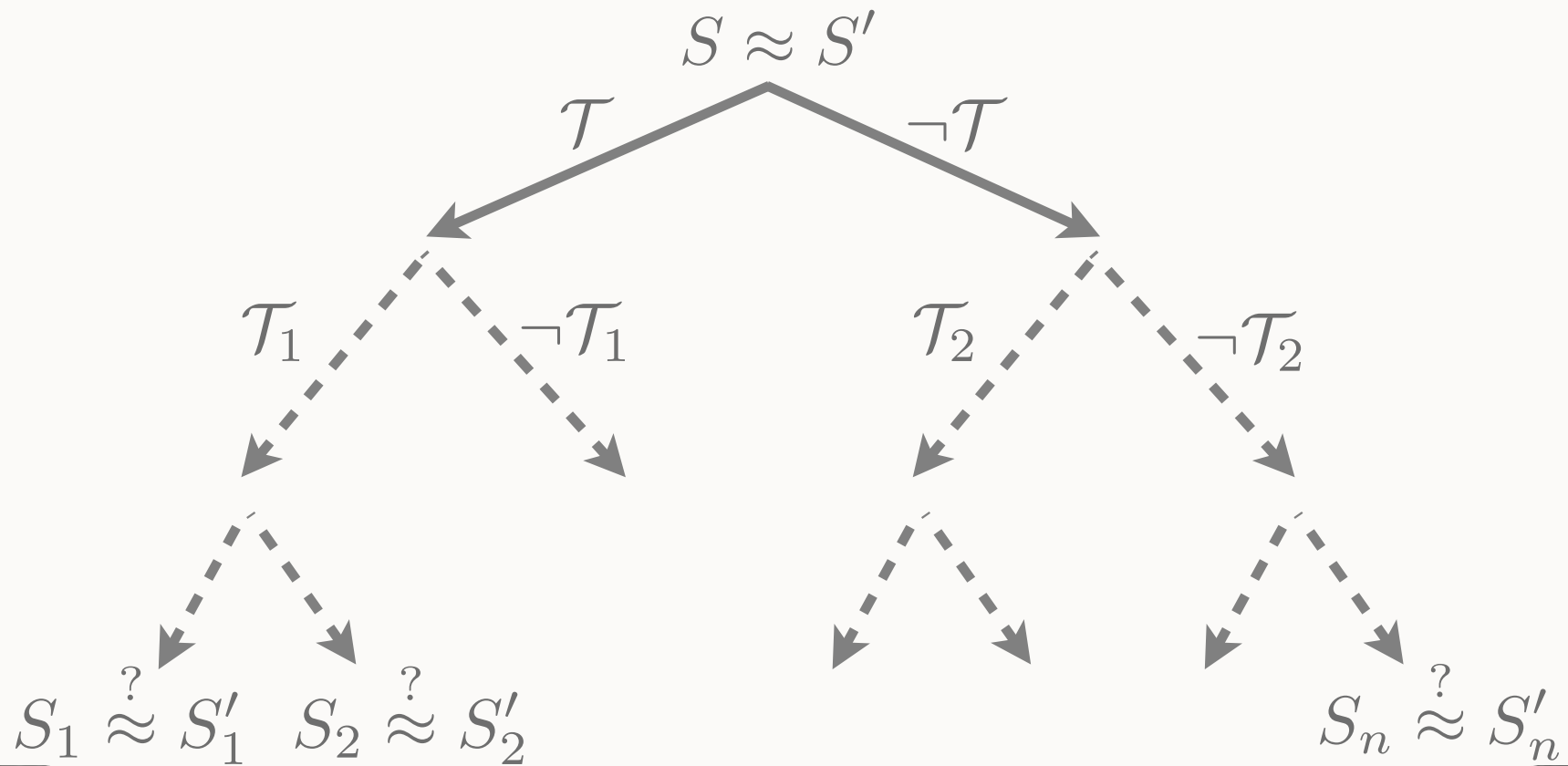
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# THE ALGORITHM

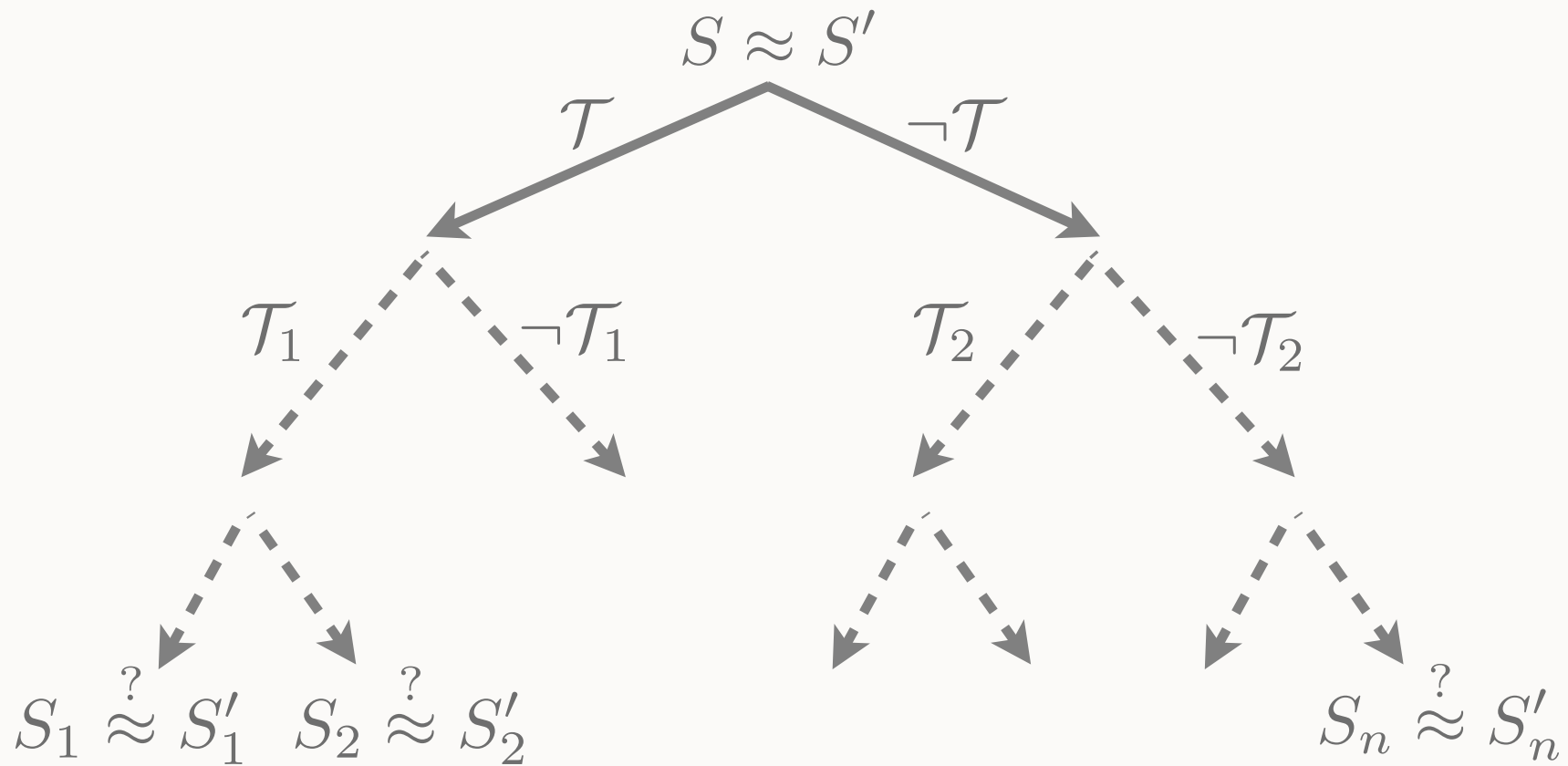
- A complete execution



The application of the rules creates a binary tree where each node is a pair of sets of constraint systems

# THE ALGORITHM

- A complete execution



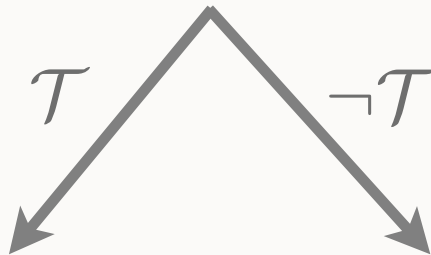
The symbolic equivalence is syntactically decided on each leaf

# THE ALGORITHM

- Example of rule : Cons

Test  $\mathcal{T} = \exists X_1, X_2$  s.t.  $X = \text{enc}(X_1, X_2)$

$$\left\{ \begin{array}{l} \dots \\ T \vdash_X \text{enc}(u_1, u_2) \\ \dots \end{array} \right.$$



$$\left\{ \begin{array}{l} \dots \\ T \vdash_{X_1} u_1 \\ T \vdash_{X_2} u_2 \\ X = \text{enc}(X_1, X_2) \\ \dots \end{array} \right.$$

$$\left\{ \begin{array}{l} \dots \\ T \vdash_X \text{enc}(u_1, u_2) \\ \text{Top}(X) \neq \text{enc} \\ \dots \end{array} \right.$$

# THE ALGORITHM

- The solved form of a constraint system
- Existence of solutions (Reachability)

$$\begin{array}{l} m_1, \dots, m_n \vdash x \\ m_1, \dots, m_n, \dots, m_{n'} \vdash y \end{array}$$

- Matching solutions (including disequations)

$$\begin{array}{l} a, b \vdash x \\ a, b, c \vdash y \\ x \neq y \end{array}$$

$$\begin{array}{l} a, b \vdash x \\ a, b, c \vdash y \\ x \neq f(y) \end{array}$$

- Static equivalence

$$\begin{array}{l} a, \{b\}_c \vdash x \\ a, \{b\}_c, c \vdash y \end{array}$$

$$\begin{array}{l} a, b \vdash x \\ a, b, c \vdash y \end{array}$$

# RESULT

Let  $(S_0, S'_0)$  be an initial pair of set of constraint systems, we have :

$(S, S')$

$(S, S')$

# RESULT

Let  $(S_0, S'_0)$  be an initial pair of set of constraint systems, we have :

If all leaves  $(S, S')$  on the tree satisfy the testing condition then  $S_0 \approx S'_0$ .

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The strategy terminates

# FUTURE WORK

## ■ Contribution

Decision procedure for trace equivalence

- Infinitely many traces are represented by symbolic constraint system
- + Protocol possibly non-determinist and with non trivial else branches
- + Private channels
- Fixed set of cryptographic primitives : symmetric and asymmetric encryption, pairing and signature
- Bounded number of sessions (no replication in the process algebra)

## ■ Future work

- Experiment shows that the implementation is not efficient enough
- More cryptographic primitives
- Link with ProVerif

# TERMINATION

- The disequations problem

$$a, b \vdash x_1$$

$$D : a, b \vdash x_2$$

$$a, b \vdash y$$

$$E : [x_1 \neq y \vee x_2 \neq a] \wedge y \neq \langle x_1, x_2, b \rangle$$

# TERMINATION

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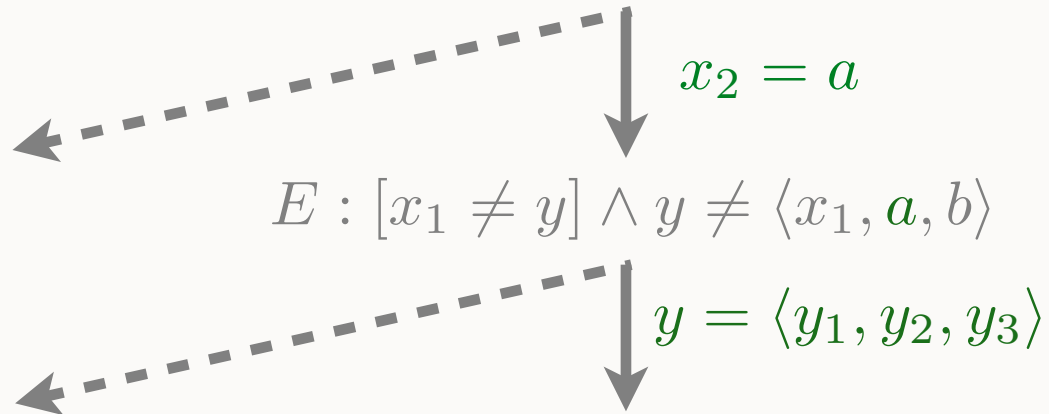


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$$E : [x_1 \neq y] \wedge y \neq \langle x_1, a, b \rangle$$



$$E : [x_1 \neq \langle y_1, y_2, y_3 \rangle] \wedge \langle y_1, y_2, y_3 \rangle \neq \langle x_1, a, b \rangle$$



# TERMINATION

- The disequations problem

$$E : [x_1 \neq y \vee x_2 \neq a] \wedge y \neq \langle x_1, x_2, b \rangle$$

$x_2 = a$

$$E : [x_1 \neq y] \wedge y \neq \langle x_1, a, b \rangle$$

$y = \langle y_1, y_2, y_3 \rangle$

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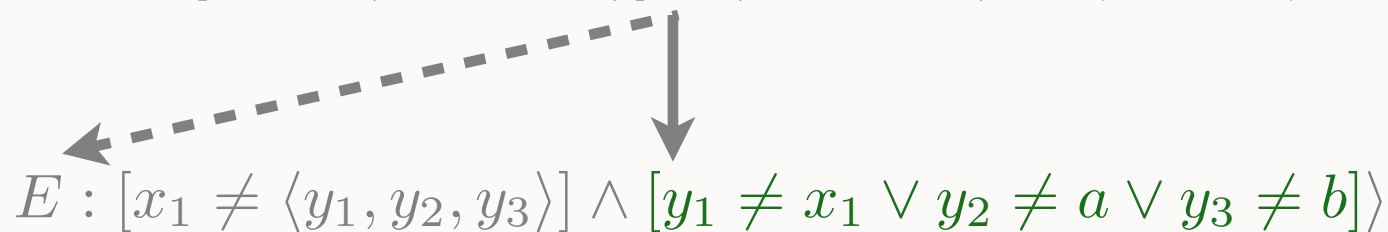
$$E : [x_1 \neq y] \wedge y \neq \langle x_1, a, b \rangle$$

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$$E : [x_1 \neq \langle y_1, y_2, y_3 \rangle] \wedge \langle y_1, y_2, y_3 \rangle \neq \langle x_1, a, b \rangle$$

$y_3 = b$



$$E : [x_1 \neq \langle y_1, y_2, y_3 \rangle] \wedge [y_1 \neq x_1 \vee y_2 \neq a \vee y_3 \neq b]$$



# TERMINATION

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$$E : [x_1 \neq \langle y_1, y_2, b \rangle] \wedge [y_1 \neq x_1 \vee y_2 \neq a]$$